

# FAULT DIAGNOSIS TOOLBOX

## – v0.13

ERIK FRISK <[frisk@isy.liu.se](mailto:frisk@isy.liu.se)>

Department of Electrical Engineering

Linköping University, Sweden

### SUMMARY

Fault Diagnosis Toolbox is a Matlab toolbox for analysis and design of fault diagnosis systems for dynamic systems, primarily described by differential equations. In particular, the toolbox is focused on techniques that utilize structural analysis, i.e., methods that analyze and utilize the model structure. The model structure is the interconnections of model variables and is often described as a bi-partite graph or an incidence matrix. Key features of the toolbox are

- Defining diagnosis models, using only model structure or full symbolic expressions.
- Diagnosability analysis - analyze a given *model* to determine which faults that can be detected and which faults that can be isolated
- Model exploration and analysis, e.g., plotting model properties, Dulmage-Mendelsohn decomposition, DAE index analysis, ...
- Finding overdetermined sets of equations (MSO sets), which are minimal sub-models that can be used to design fault detectors
- Sensor placement - determine minimal sets of sensors needed to be able to detect and isolate faults
- Code generation (Matlab and C) for residual generators. Two different types of residual generators are supported, sequential residual generators based on a matching in the model structure graph, and observer based residual generators.

The toolbox relies on the object-oriented functionality of the Matlab language and is freely available under a MIT license. The latest version can always be downloaded from our website at <http://www.fs.isy.liu.se/Software/FaultDiagnosisToolbox/> and links to relevant publications can be found also at our list of publications <http://www.fs.isy.liu.se/Publications>.

## CONTRIBUTORS

The following people has contributed with code

- Erik Frisk, Department of Electrical Engineering, Linköping University, Sweden.
- Mattias Krysanter, Department of Electrical Engineering, Linköping University, Sweden.
- Daniel Jung, Department of Electrical Engineering, Linköping University, Sweden.

## CONTENTS

1	Introduction and overview	4
1.1	Reference literature . . . . .	5
1.2	Downloading and installation . . . . .	5
1.3	Terms of usage . . . . .	6
2	Defining models	6
2.1	Defining a structural model . . . . .	7
2.1.1	Defining the model using incidence matrices . . . . .	7
2.1.2	Defining the model using variable names . . . . .	7
2.1.3	Defining dynamic models . . . . .	9
2.2	Defining symbolic models . . . . .	11
2.2.1	Conditional constraints . . . . .	12
2.2.2	Using external functions . . . . .	12
3	Dulmage-Mendelsohn decomposition	12
4	Analysis of overdetermined equations	13
5	Diagnosability analysis	15
6	Test Selection	15
6.1	A minimal hitting set approach . . . . .	16
6.2	Random Forest based Approach . . . . .	16
7	Sensor placement analysis	16
8	Residual generator design	18
8.1	Sequential residual generator design . . . . .	18
8.2	Generating and compiling generated C code . . . . .	21
8.3	Observer based residual generator design . . . . .	21
9	Use case	23
9.1	Model definition . . . . .	24
9.2	Isolability analysis . . . . .	25
9.3	Find overdetermined set of equations . . . . .	26
9.4	Design residual generators . . . . .	27
9.5	Isolability properties of residual generators . . . . .	28
9.6	Simulation results . . . . .	29
A	Summary of class methods	32
B	Compile C++ implementations	34
B.1	Minimal Hitting Set . . . . .	34
B.2	MSO algorithm . . . . .	34
C	Generated code in use-case	35
C.1	ResGen1 . . . . .	36
C.2	ResGen2 . . . . .	37
C.3	ResGen3 . . . . .	37
C.4	ResGen4 . . . . .	39
D	Index of keywords and methods	40

## 1 INTRODUCTION AND OVERVIEW

This toolbox covers a set of methods and functionality for fault diagnosis of dynamic systems described by differential (or static) equations. The field of fault diagnosis is wide and there are many available methods described in the literature. This toolbox focuses on techniques from the Automatic Control community (Safeprocess) and some from the AI field (DX). In particular, techniques related to *structural analysis* is covered because they are particularly suited to automate in a computer tool. This manual is not intended as a book on diagnosis or structural methods and some material covered requires knowledge outside this text. See Section 1.1 for some pointers to relevant literature.

This manual will not cover all the details, options, and outputs of all available methods, instead it will cover typical uses. The outline of the manual is that in Section 2, it will be covered how to define models, and then in Sections 3-8, different analysis and design techniques included in the toolbox will be covered. Section 9 describes a use case, beginning with a model definition and all the way to simulation of automatically generated residual generators. The directory examples in the source distribution include a number of use cases, including the one covered in Section 9.

The toolbox requires Matlab v7.6 (R2008a) or later for the object-oriented functionality and for symbolic math functionality, the Symbolic Toolbox version 7.14 (2012a) or later is required<sup>1</sup>. The fault diagnosis toolbox utilizes the object-oriented functionality of the Matlab language and the main class is `DiagnosisModel`. The most detailed documentation of the methods can be found in Matlab and to start the help browser, write

```
1 >> doc DiagnosisModel
```

It is also possible to list all available methods by

```
1 >> methods DiagnosisModel
2
3 Methods for class DiagnosisModel:
4
5 AddEquations           Matching
6 AddSensors             MeasurementEquations
7 AlgebraicVariables     ObserverResGen
8 BipartiteToLaTeX       Pantelides
9 CompiledMHS            PlotDM
10 CompiledMSO            PlotMatching
11 DetectabilityAnalysis  PlotModel
12 DiagnosisModel         PossibleSensorLocations
13 DifferentialConstraints Redundancy
14 DynamicVariables       RemoveFaultVariables
15 FSM                    ReplaceEquations
16 GenSimulationModel     SensorLocationsWithFaults
17 IsDynamic              SensorPlacementDetectability
18 IsHighIndex            SensorPlacementIsolability
19 IsLowIndex             SeqResGen
20 IsObservable           StateVariables
21 IsPSO                  Structural
22 IsStatic               SubModel
23 IsolabilityAnalysis    TestSelection
24 IsolabilityAnalysisArrs copy
25 IsolabilityAnalysisFSM srank
```

<sup>1</sup> The toolbox is primarily developed and tested under v8.4 (R2014b) and later versions of Matlab/Symbolic Toolbox.

```

26 Lint                ne
27 LumpDynamics       nf
28 MSO                 nx
29 MSOCausalitySweep nz
30 MTES
31 MTESRedundancy
32
33 Methods of DiagnosisModel inherited from handle.

```

To obtain help for a particular method, here for example the `PlotDM` method, write

```

1 >> help DiagnosisModel.PlotDM
2 PlotDM Plots Dulmage–Mendelsohn decomposition of model structure
3
4 [row,col,psodecomp] = model.PlotDM( options )
5
6 Plots a Dulmage–Mendelsohn decomposition, originally described in
7 Dulmage, A. and Mendelsohn, N. "Coverings of bipartite graphs."
8 Canadian Journal of Mathematics 10.4 (1958): 516–534.
9
10 By default, the Dulmage_Mendelsohn decomposition is plotted for the
11 incidence matrix for the unknown variables.
12
13 Options can be given as a number of key/value pairs
14
15 Key      Value
16 eqlclass If true, perform canonical decomposition of M+ and
17           plot equivalence classes
18
19           For further details on the canonical decomposition
20           of the M+ part of the structure, see Chapter 4 in
21           "Design and Analysis of Diagnosis Systems Using Structural
22           Methods", PhD thesis, Mattias Krysander, 2006.
23 fault    If true, indicates fault equations in canonical
24           decomposition of M+
25
26 submodel Array of equation indices corresponding to submodel.
27
28 Outputs:
29 row      – row permutation used in the plot
30 col      – column permutation used in the plot
31 psodecomp – result of psodecomposition of the M+ part
32
33 Example:
34 model.PlotDM( 'eqlclass', true, ' fault ', true )

```

### 1.1 Reference literature

Our publications on structural methods (all should not be included): [17, 18, 6, 10, 5, 11, 15, 16]

Other (include more): [1]

### 1.2 Downloading and installation

The latest version of the package can always be obtained from <http://www.fs.isy.liu.se/Software/FaultDiagnosisToolbox/> and the installation is very simple:

1. Uncompress the tar.gz/zip-file
2. Add the src directory to the Matlab-path
3. (optional) There are C++ implementations of some computationally expensive algorithms. It is not necessary to compile these for the toolbox to work, there are Matlab implementations directly installed. However, with the compiled versions there might be significant increases in performance.

For linux and Mac OS X there are pre-compiled versions and to install, copy the suitable files from the binaries directory into the src directory. See Appendix B for details how to compile C++ sources.

In the archive there is also a manual (this document) and a directory with a few example usages of toolbox functionality. The toolbox requires Matlab v7.6 (R2008a) or newer and for full functionality it requires access to the symbolic math toolbox.

### 1.3 Terms of usage

The toolbox is free for anyone to use, it is distributed under the MIT License (MIT) (<http://opensource.org/licenses/MIT>). If you encounter bugs, have comments, or suggestions, please contact Erik Frisk <[erik.frisk@liu.se](mailto:erik.frisk@liu.se)>. If you use the toolbox in your scientific work, please cite the toolbox (not yet published) *and* the corresponding methodological publication. The relevant publication is indicated in the help text for the class methods and functions.

## 2 DEFINING MODELS

A first step in using the toolbox is to define the model object. There are two types of model specifications

- *structural* model
- *symbolic* model

A structural model only contains information about model structure and does not need specifications on the underlying symbolic expressions. Many of the analysis methods can be applied to structural models and it is mainly the residual generation methods that need the symbolic expressions. When defining a symbolic model, the toolbox automatically computes the model structure.

To illustrate, the following small example will be used.

$$\begin{aligned}
 e_1 : \dot{x}_1 &= -c_1x_1 + x_2 + x_5 \\
 e_2 : \dot{x}_2 &= -c_2x_2 + x_3 + x_4 \\
 e_3 : \dot{x}_3 &= -c_3x_3 + x_5 + f_1 + f_2 \\
 e_4 : \dot{x}_4 &= -c_4x_4 + x_5 + f_3 \\
 e_5 : \dot{x}_5 &= -c_5x_5 + u + f_4 \\
 e_6 : y_1 &= x_1 \\
 e_7 : y_2 &= x_2 \\
 e_8 : y_3 &= x_3
 \end{aligned} \tag{1}$$

The variables  $x_i$  are the unknown states,  $y_i$  measurement signals,  $u$  known control input,  $f_i$  the faults, and  $c_i$  known model parameters. The model structure is then given by Table 1. Here, the dynamics are *lumped*, i.e., the variables are considered as *signals* and it is not important if a variable is differentiated or not. See [5] for some further discussion on this and for dynamic models, see Section 2.1.3 for further discussion where it is shown how to explicitly define non-lumped dynamic models.

Table 1: Model structure of example model (1).

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$y_1$	$y_2$	$y_3$	$u$	$f_1$	$f_2$	$f_3$	$f_4$
$e_1$	X	X			X								
$e_2$		X	X	X									
$e_3$			X		X					X	X		
$e_4$				X	X							X	
$e_5$					X				X				X
$e_6$	X					X							
$e_7$		X					X						
$e_8$			X					X					

## 2.1 Defining a structural model

There are two ways of defining a structural model; either the incidence matrices are given directly or the variable names for each equation are specified.

### 2.1.1 Defining the model using incidence matrices

To define this model structure in the toolbox, i.e., creating the model object, the first important function call is `DiagnosisModel`. To use this function, define a structure with the model specification and then call `DiagnosisModel` with the structure as argument. The model specification has 4 important fields

- `type` - when specifying a model using the incidence matrices, this should be the string `MatrixStruc`
- `X` - incidence matrix for the unknown variables
- `F` - incidence matrix for the faults
- `Z` - incidence matrix for the known variables

In Matlab, this becomes

```

1 modelDef.type = 'MatrixStruc';
2 modelDef.X = [1 1 0 0 1; 0 1 1 1 0; 0 0 1 0 1; 0 0 0 1 1; 0 0 0 0 1; ...
3             1 0 0 0 0; 0 1 0 0 0; 0 0 1 0 0];
4 modelDef.F = [0 0 0 0; 0 0 0 0; 1 1 0 0; 0 0 1 0; ...
5             0 0 0 1; 0 0 0 0; 0 0 0 0; 0 0 0 0];
6 modelDef.Z = [0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0; ...
7             0 0 0 1; 1 0 0 0; 0 1 0 0; 0 0 1 0];

```

The variable names are by default  $x_i$ ,  $z_i$  and  $f_i$ . To specify the variable names explicitly, add field names `x`, `z`, or `f` respectively. For example, to specify the known variable names as in the model (1), add

```

1 modelDef.z = {'y1','y2','y3','u'};

```

After the model specification is done, create the model object by running

```

1 model = DiagnosisModel(modelDef);
2 model.name = 'Example model';

```

where also a name for the model can be specified (optional).

### 2.1.2 Defining the model using variable names

Defining incidence matrices is prone to errors and a more convenient way to define model the model structure is by only providing variable names. Again, a model structure is defined with 5 important fields

- type - when specifying a model using the incidence matrices, this should be the string `VarStruc`
- x - cell array with unknown variable names
- f - cell array with fault variable names
- z - cell array with known variable names
- reIs - a cell array describing the variables in each equation.

For the model (1), this model specification, which is equivalent to the model definition above, becomes

```

1 modelDef.type = 'VarStruc';
2 modelDef.x = {'x1','x2','x3','x4','x5'};
3 modelDef.z = {'y1','y2','y3','u'};
4 modelDef.f = {'f1','f2','f3','f4'};
5 modelDef.reIs = {'x1','x2','x5'},
6   {'x2','x3','x4'},{'x3','x5','f1','f2'},...
7   {'x4','x5','f3'},{'x5','f4','u'},...
8   {'y1','x1'},{'y2','x2'},{'y3','x3'};
9 model = DiagnosisModel(modelDef);
10 model.name = 'Example model';

```

Now that the model is defined, we can try a few simple operations on the model object. For example, the model structure can be plotted using the class method `PlotModel`. The command

```
1 model.PlotModel()
```

will result in Figure 1. The class method `Lint` does some basic validity check on the

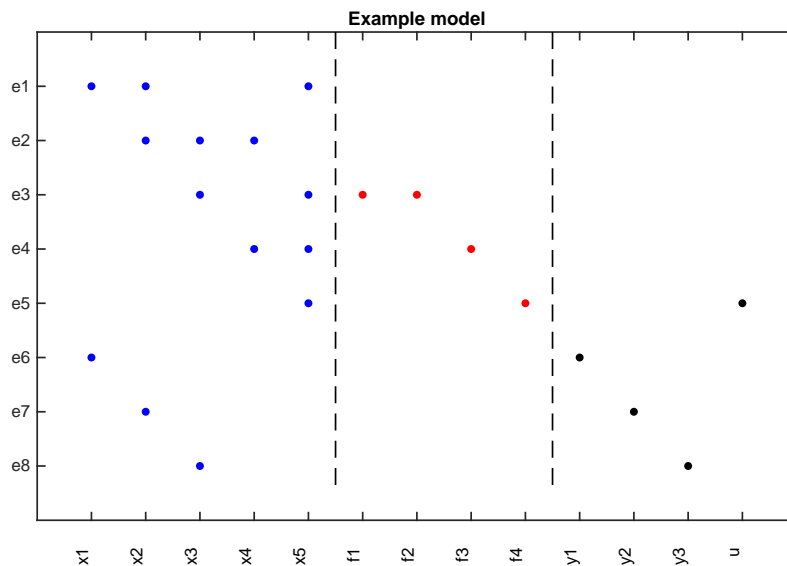


Figure 1: Result of class method `PlotModel`.

model definition, e.g.,

```

1 >> model.Lint()
2 Model: Example model
3
4 Variables
5   5 unknown variables
6   4 known variables

```



```

7 4 fault variables
8 8 equations, including 0 differential constraints
9 Degree of redundancy: 3
10
11 Model validation finished with 0 errors and 0 warnings

```

The model structure can also be illustrated using a bi-partite graph, the class method `BipartiteToLaTeX` generates  $\text{\LaTeX}$  code generating a figure that can be directly type-set using the  $\text{\LaTeX}$ -engine. For example

```

1 model.BipartiteToLaTeX('bipartite.tex', 'faults', true, 'shortnames', true);

```

generates code for Figure 2.

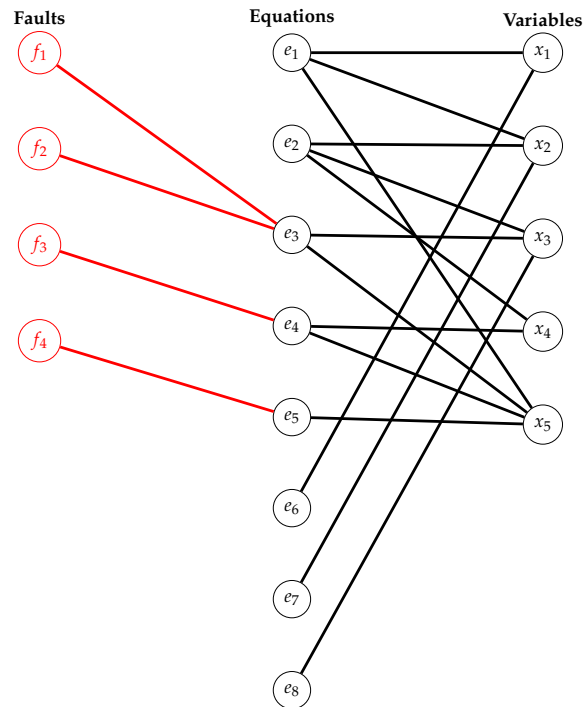


Figure 2: Result of class method `BipartiteToLaTeX`.

As a final note it is important to understand that the `DiagnosisModel` class is a *handle*<sup>2</sup> class in Matlab. This means that the variable `model` above is a *reference* to the object. Thus, making

```

1 model2=model; % Warning! No new object

```

does *not* make a copy of the object, it merely stores another reference to the *same* object. To get a new copy, use the `copy` class method as

```

1 model2 = model.copy(); % Safe, new object created

```

### 2.1.3 Defining dynamic models

To make systematic analysis of some dynamic properties of the model, there is a need to explicitly state the dynamic variables. The way to do this is in the toolbox is to introduce some new variables and explicit *differential*-constraints. Thus, for model (1), introduce variables  $dx_i$  for the differentiated variables, and add 5

<sup>2</sup> See <http://www.mathworks.com/help/matlab/handle-classes.html> for details.

differential constraints, one for each state-variable, explicitly connecting the differentiated variable with the non-differentiated. Thus, a model description, equivalent to (1), is

$$\begin{aligned}
 e_1 : dx_1 &= -c_1x_1 + x_2 + x_5 & e_9 : dx_1 &= \frac{d}{dt}x_1 \\
 e_2 : dx_2 &= -c_2x_2 + x_3 + x_4 & e_{10} : dx_2 &= \frac{d}{dt}x_2 \\
 e_3 : dx_3 &= -c_3x_3 + x_5 + f_1 + f_2 & e_{11} : dx_3 &= \frac{d}{dt}x_3 \\
 e_4 : dx_4 &= -c_4x_4 + x_5 + f_3 & e_{12} : dx_4 &= \frac{d}{dt}x_4 \\
 e_5 : dx_5 &= -c_5x_5 + u + f_4 & e_{13} : dx_5 &= \frac{d}{dt}x_5 \\
 e_6 : y_1 &= x_1 \\
 e_7 : y_2 &= x_2 \\
 e_8 : y_3 &= x_3
 \end{aligned} \tag{2}$$

and here the differential-constraints  $e_9$ - $e_{13}$  is explicit. Model (2) can be defined in the toolbox using variable names just as before, but using the function `DiffConstraint` to define the differential constraints. Matlab code to define the dynamic model then becomes

```

1 modelDef.type = 'VarStruc';
2 modelDef.x = {'dx1','dx2','dx3','dx4','dx5', 'x1','x2','x3','x4','x5'};
3 modelDef.z = {'y1','y2','y3','u'};
4 modelDef.f = {'f1','f2','f3','f4'};
5 modelDef.rels = {'dx1','x1','x2','x5'}, {'dx2','x2','x3','x4'},...
6   {'dx3','x3','x5','f1','f2'}, {'dx4','x4','x5','f3'},...
7   {'dx5','x5','f4','u'}, {'y1','x1'}, {'y2','x2'}, {'y3','x3'},...
8   DiffConstraint('dx1','x1'), DiffConstraint('dx2','x2'),...
9   DiffConstraint('dx3','x3'), DiffConstraint('dx4','x4'),...
10  DiffConstraint('dx5','x5');
11 model = DiagnosisModel( modelDef );

```

Calling the `PlotModel` method results in Figure 3. This way of defining the dy-

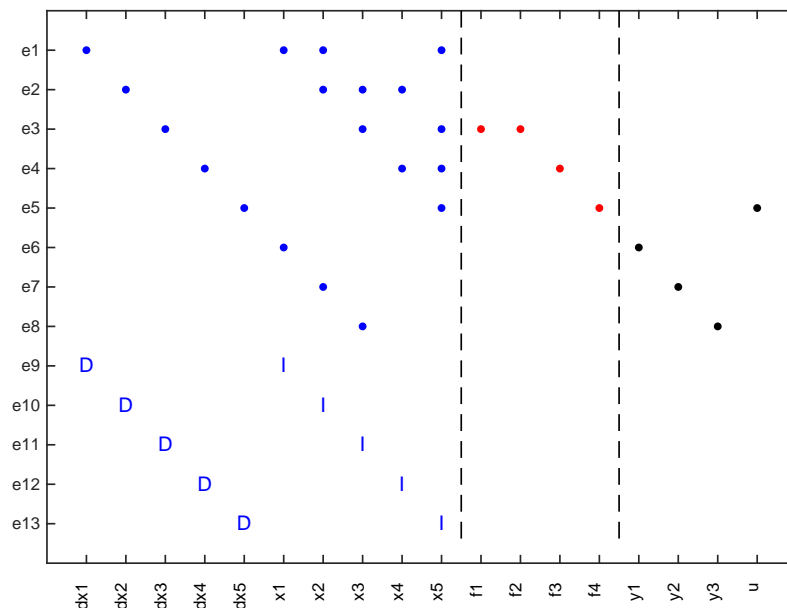


Figure 3: Result of calling the `PlotModel` method for the model (2). Note the I and D that indicate differentiated/integrated variable relation.

namics is not necessarily optimal since it may introduce unnecessary variables and also may affect the index of the model. However, this forces the model to be in semi-explicit form

$$\dot{x}_1 = g_1(x_1, x_2) \quad (3a)$$

$$0 = g_2(x_1, x_2) \quad (3b)$$

which makes further analysis significantly easier. For example, the semi-explicit form (3) is (locally) low differential index [2] if and only if

$$\frac{\partial g_2}{\partial x_2}$$

is full rank. For a fully implicit DAE, additional analysis is needed, for example utilizing the algorithm in [13].

## 2.2 Defining symbolic models

Defining a symbolic model, i.e., specifying the symbolic expressions for the model constraints, makes it possible to make the same structural analyses as for the structural models and, in addition, generate code for residual generators. The Symbolic Math Toolbox for Matlab<sup>3</sup> is required for this to work.

To specify the model, create a model structure with type `Symbolic` and define the model variables as before. Also add names of model parameters. In the case of model (2), this looks like

```
1 modelDef.type = 'Symbolic';
2 modelDef.x = {'dx1','dx2','dx3','dx4','dx5', 'x1','x2','x3','x4','x5'};
3 modelDef.z = {'y1','y2','y3','u'};
4 modelDef.f = {'f1','f2','f3','f4'};
5 modelDef.parameters = {'c1','c2','c3','c4','c5'};
```

To specify values for parameters, not necessary, include a struct with the parameter values as

```
1 param_values.c1 = 1.0;
2 param_values.c2 = 2.0;
3 param_values.c3 = 3.0;
4 param_values.c4 = 4.0;
5 param_values.c5 = 5.0;
```

and then include in the model definition as

```
1 modelDef.parameter_values = param_values;
```

The next step is to make all model variables and parameters symbolic. This is achieved with

```
1 syms(modelDef.x{:})
2 syms(modelDef.f{:})
3 syms(modelDef.z{:})
4 syms(modelDef.parameters{:})
```

Now that all variables and parameters are symbolic, the relations of the model can be written down and the model object created as before.

```
1 modelDef.rels = {...
2   dx1 == -c1*x1+x2+x5,...
3   dx2 == -c2*x2+x3+x4,...
```

<sup>3</sup> <http://www.mathworks.com/products/symbolic/>

```

4 dx3 == -c3*x3 + x5 + f1 + f2 ,...
5 dx4 == -c4*x4+x5+f3,...
6 dx5 == -c5*x5 + u + f4 ,...
7 y1 == x1, y2 == x2, y3 == x3, ...
8 DiffConstraint(' dx1 ',' x1 '),DiffConstraint(' dx2 ',' x2 '),...
9 DiffConstraint(' dx3 ',' x3 '),DiffConstraint(' dx4 ',' x4 '),...
10 DiffConstraint(' dx5 ',' x5 ');
11 model = DiagnosisModel( modelDef );
12 model.name = 'Example model';

```

The differential constraints are added, as before, using the directive `DiffConstraint`.

To tidy up, the symbolic variables can be cleared from the workspace using the commands

```

1 % clear temporary variables from workspace
2 clear( modelDef.x{:} )
3 clear( modelDef.f{:} )
4 clear( modelDef.z{:} )
5 clear( modelDef.parameters{:} )

```

When the symbolic model is specified, the model structure is automatically computed and all the analysis/design tools utilizing the model structure can be directly applied, in addition to some new methods operating on symbolic expressions. In particular this applies to residual generation described in Section 8.

### 2.2.1 Conditional constraints

Not yet fully implemented. At the moment, define an external function as in Section 2.2.2 for this functionality.

### 2.2.2 Using external functions

It is often the case in more complex models that there are external functions like look-up tables. Such functions can be used when defining the model. For example, consider that the model has a function for  $\eta_c$  with arguments as

$$\eta_c(\eta_{c,\max}, \eta_{c,\min}, X, Q)$$

Then, defining the function to be a symbolic function as

```

1 syms eta_c_fun(eta_cmax,eta_cmin,X,Q)

```

and then the function can be used as any other. Of course, the symbolic toolbox will not be able to do symbolic manipulations of that particular function and the causality of equations using that function will be uniquely determined by the model formulation.

## 3 DULMAGE-MENDELSON DECOMPOSITION

When doing any sorts of structural analysis for fault diagnosis, the Dulmage-Mendelsohn decomposition [4] is a very useful tool [1]. Given a structural model, by proper and well defined reordering of variables and equations, a structure graph can always be transformed into the form shown in Figure 4. If  $X$  is a structure matrix, the command

```

1 dm = GetDMParts( X );

```

is a simple wrapper around the `dmperm` command in Matlab, which computes the Dulmage-Mendelsohn decomposition. The variable `dm` is a structure with 7 fields:

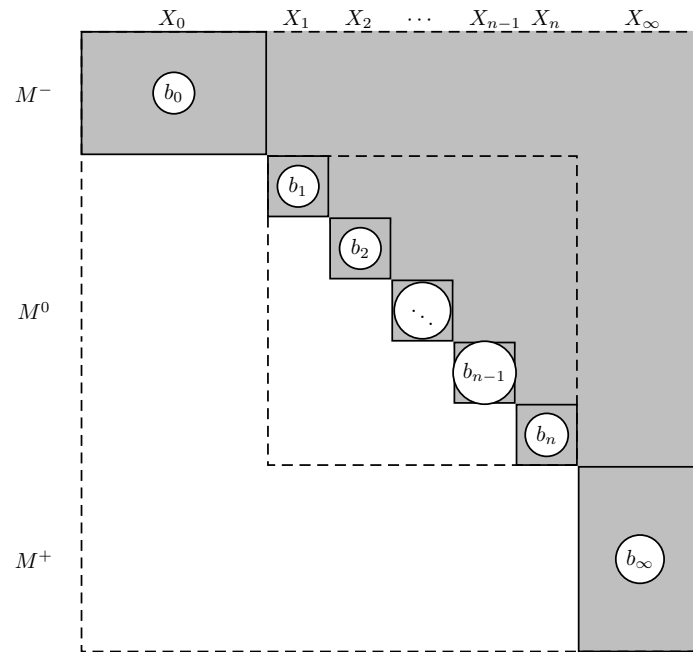


Figure 4: Dulmage-Mendelsohn decomposition

- $M_m$  - structure defining the rows and columns of the under-determined part  $M^-$ .
- $M^0$  - cell array with structures defining the Hall components in  $M^0$
- $M_p$  - structure defining the rows and columns of the over-determined part  $M^+$ .
- $M^0_{eqs}$  - collection of all rows in  $M^0$
- $M^0_{vars}$  - collection of all columns in  $M^0$
- $rowp$  - original row permutation
- $colp$  - original column permutation

For fault diagnosis, there is a particular decomposition of the overdetermined part that is of particular interest. The decomposition is defined in [11] and can be computed using the `PSODecomposition` command. There is also a class method that can plot the Dulmage-Mendelsohn decomposition of the model structure in an informative way. For this method, there are two options that can be activated, perform the decomposition of the over-determined part of the model, and indicate which equations that are influenced by faults. This is particularly important in diagnosability analysis. Below is a method call with both options activated,

```
model.PlotDM('eqclass', true, 'fault', true)
```

and the result, for the three-tank model in [6], is shown in Figure 5.

## 4 ANALYSIS OF OVERDETERMINED EQUATIONS

Overdetermined parts of a model is highly interesting for fault diagnosis since these are the parts with redundancy and thereby possible to use for fault diagnosis. In the toolbox, there are two main class methods: `MSO` and `MTES`. The set of Minimally Structurally Overdetermined (MSO) sets of equations are subset minimal sets of equations with redundancy. Implemented in the toolbox is the algorithm from [11]. It is straightforward to apply. Given a model object `model`, the command

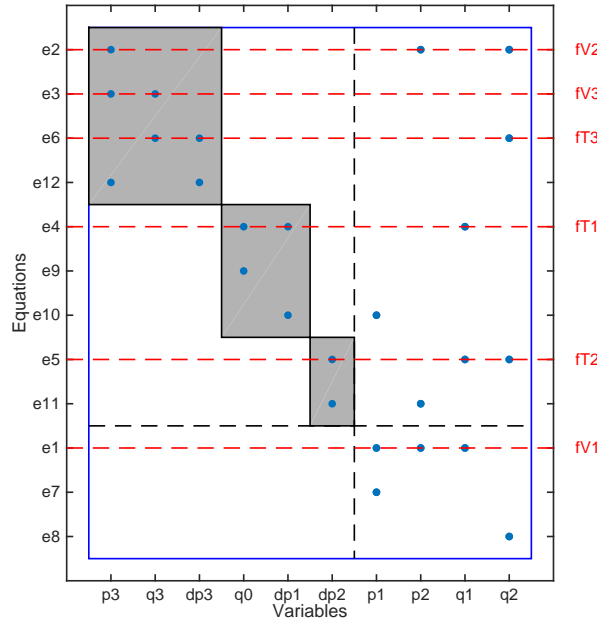


Figure 5: Dulmage-Mendelsohn decomposition, with equivalence class decomposition, and fault equation indication activated.

```
msos = model.MSO());
```

the set of all MSO sets are computed. Please beware that the cardinality of the set of MSO sets is exponential in the degree of redundancy of the model. Therefore, when the redundancy gets high enough the computational complexity of the algorithm becomes very high. The notion of MSO is related to other works concerning overdetermined sets of equations, see [12] for further discussions and similarities. Instructions to install compiled C++ version of the MSO algorithm for increased performance can be found in Appendix B.2.

For the model (2), there are 11 MSO sets where one for example is  $MSO_1 = \{3, 5, 8, 11, 13\}$  which means that the equations

$$\begin{aligned} e_3 : dx_3 &= -c_3x_3 + x_5 + f_1 + f_2 & e_{11} : dx_3 &= \frac{d}{dt}x_3 \\ e_5 : dx_5 &= -c_5x_5 + u + f_4 & e_{13} : dx_5 &= \frac{d}{dt}x_5 \\ e_8 : y_3 &= x_3 \end{aligned}$$

are overdetermined and can be used to design a residual generator. For example, based on the equations above one can derive the ARR

$$r = \dot{y}_3 + (c_3 + c_5)\dot{y}_3 + c_3c_5y_3 - u$$

or the observer based residual generator

$$\begin{aligned} \hat{x}_3 &= -c_3\hat{x}_3 + \hat{x}_5 + K_1(y_3 - \hat{x}_3) \\ \hat{x}_5 &= -c_5\hat{x}_5 + u + K_2(y_3 - \hat{x}_3) \\ r &= y_3 - \hat{x}_3 \end{aligned}$$

where  $K_1$  and  $K_2$  are observer gains to ensure observer stability.

As mentioned above, the size of the set of MSO sets is exponential in the redundancy of the model and therefore may not be applicable to high-redundancy problems. For this reason, a second type of overdetermined sets of equations might be of interest, Minimal Test Equation Support (MTES), defined in [9]. These do not have the as severe complexity issues as the MSO sets and the class method is called in a similar way using the method MTES

```
1 mtes = model.MTES();
```

## 5 DIAGNOSABILITY ANALYSIS

A set of methods for analyzing diagnosability of a *model* or a set of *residual generators* are available. Here, diagnosability means to analyze which faults that are structurally detectable and structurally isolable. Basic definitions on detectability and isolability used in the toolbox can be found in [10, 6].

For a basic detectability analysis of a given *model*, use the class method `DetectabilityAnalysis` as

```
1 [df,ndf] = model.DetectabilityAnalysis();
```

The `df` output is the set of detectable faults and `ndf` the set of non-detectable faults.

Similarly, to plot a fault isolability analysis of the *model* use the class method `IsolabilityAnalysis` as

```
1 model.IsolabilityAnalysis( );
```

With no output arguments, the method plots the analysis. It is possible to restrict the analysis to causality assumptions [6] which here means that the analysis can be done in *derivative causality*, *integral causality*, or *mixed causality*. The mixed causality is the default if no causality assumption is specified. To explicitly specify the causality assumption, write

```
1 model.IsolabilityAnalysis( 'causality', 'der');
2 model.IsolabilityAnalysis( 'causality', 'int');
3 model.IsolabilityAnalysis( 'causality', 'mixed');
```

See Figures 10 and 11 for example outputs. The interpretation is that with a non-zero element at position  $(i, j)$  means that fault in column  $j$  can not be isolated from fault in row  $i$ .

It is also possible to do analysis on a set of ARRs, represented as sets of equations to be used to design residual generators. For example, to see what is the isolability properties of a diagnosis system based on MSO 1 and 3 (just example numbers), use the class method `IsolabilityAnalysisArrs` as

```
1 msos = model.MSO();
2 model.IsolabilityAnalysisARR( msos([1,3]) );
```

It is also possible to obtain the fault sensitivity matrix (FSM) using the class method `FSM` as

```
1 FSM = model.FSM( Msos([1,3]) );
```

and then perform the analysis on the fault signature matrix using the class method `IsolabilityAnalysisFSM` as

```
1 model.IsolabilityAnalysisFSM( FSM );
```

## 6 TEST SELECTION

With a set of MSOs, or the corresponding fault signature matrix (FSM), it is an interesting problem how to select a subset of tests that achieves required fault isolability

performance. In general, not all possible tests are needed and often substantially less. The toolbox currently supports a simple minimal hitting set based approach to selecting tests and a data-driven approach based on Random Forest machine learning classifiers.

### 6.1 A minimal hitting set approach

The minimal hitting-set approach is implemented in the class method `TestSelection`, see [17] for further discussion on this approach. The following call finds all subset minimal sets of tests, based on the fault signature matrix `FSM`, such that maximal fault isolability is possible.

```
ts = model.TestSelection(FSM);
```

It is also possible to use the set of MSOs directly

```
ts = model.TestSelection(msos);
```

The above problem has poor complexity properties and can very quickly become intractable and therefore other methods are available. For example, to choose an approximate hitting set approach called *aminc*, which will finish fast but not guarantee a minimal solution, call

```
ts = model.TestSelection(msos, 'method', 'aminc');
```

### 6.2 Random Forest based Approach

The minimal hitting-set approach is implemented, not in a class method since the test selection is not tightly coupled to a model but instead a data set. Instead, the method is implemented in the `RandomForestTestSelection`, see [7] for details. In the example suite there is the engine data example from [7] implemented. For further details, and a Python implementation, see also the public git repository <https://gitlab.liu.se/erifr93/MLTestSelection>.

## 7 SENSOR PLACEMENT ANALYSIS

Sensor placement, or possible sensor selection, is the task of choosing a set of sensors such that diagnosis specifications are possible to reach. This toolbox implements the methods in [10].

A first step, before any analysis is possible, the set of possible sensor locations must be defined. As a principle of the approach, sensors measure single variables among the unknown variables. Thus, as is common, some sensors measure a function of the unknown variables  $x$ , and possibly known variables  $z$ , add a new variable and equation to the model

$$x_{new} = f(x)$$

Then,  $x_{new}$  is the new possible sensor location. Possible sensor locations is specified using the class method `PossibleSensorLocations`. For example, below it is specified that the first 15 unknown variables in the model is possible sensor locations.

```
model.PossibleSensorLocations(model.x(1:15));
```

Sensor locations can be specified by name, as above, or just simply indices into the set of unknown variables. For example, if positions 1, 2, 5, and 7 are possible sensor locations, use

```
model.PossibleSensorLocations([1, 2, 5, 7]);
```



Also these new sensors may fail and if we want to include that into the analysis, there is a need to specify sensor locations where added sensors may fail. This is done using the class method `SensorLocationsWithFaults`. For example, if all new sensors may fail, use

```
1 model.SensorLocationsWithFaults( model.x );
```

Now that possible sensor locations have been specified, to compute all minimal sensor sets that achieves detectability of the faults, use the method `SensorPlacementDetectability` as

```
1 sDet = model.SensorPlacementDetectability();
2 sDet{:}
3 ans =
4 'Pz' 'Q'
5
6 ans =
7 'Pz' 'Qv' 'Qv3'
```

In this case, there are two minimal solutions where the first one is to measure the variables  $\{P_z, Q\}$  and the second is  $\{P_z, Q_v, Q_{v3}\}$ <sup>4</sup>

If we want to add the first set of sensors, call `AddSensors`

```
1 model.AddSensors( sDet{1} );
```

This command modifies the model `model`. In case you want a new object, with the new sensors, without modifying the original model, write

```
1 model2 = model.AddSensors( sDet{1} );
```

The isolability properties, as described in Section 5, after adding the detectability sensors is shown in Figure 6. It is clear that all faults are detectable, but the isolation performance is far from ideal.

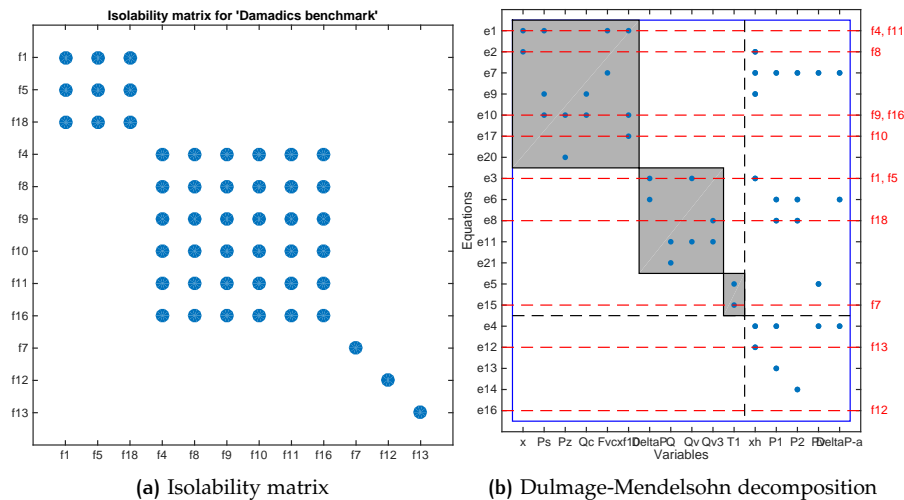


Figure 6: Isolability properties after adding the detectability sensors.

If the isolability in Figure 6 is not sufficient, we can instead call the class method `SensorPlacementIsolability` to find sets of sensors that not only detects fault but also makes fault isolation possible. For the same example as above, the Matlab call is

```
1 sIsol = model.SensorPlacementIsolability();
2 model3 = model.AddSensors( sIsol{1} );
```

<sup>4</sup> The example is taken from paper [10] and `Damadics.m` can be found in the `examples` directory.

There are 6 solutions, each involving 5 sensors.

```

1 >> sIsol
2
3 sIsol =
4
5 {1x5 cell} {1x5 cell} {1x5 cell} {1x5 cell} {1x5 cell} {1x5 cell}

```

Here, again, the first solution is added to the model and again, using the diagnosability analysis methods from Section 5, results in Figure 7. Here it is clear that, except for faults entering the model in the same equations ( $\{f_1, f_5\}$ ,  $\{f_4, f_{11}\}$ , and  $\{f_9, f_{16}\}$ ), full isolability is achieved.

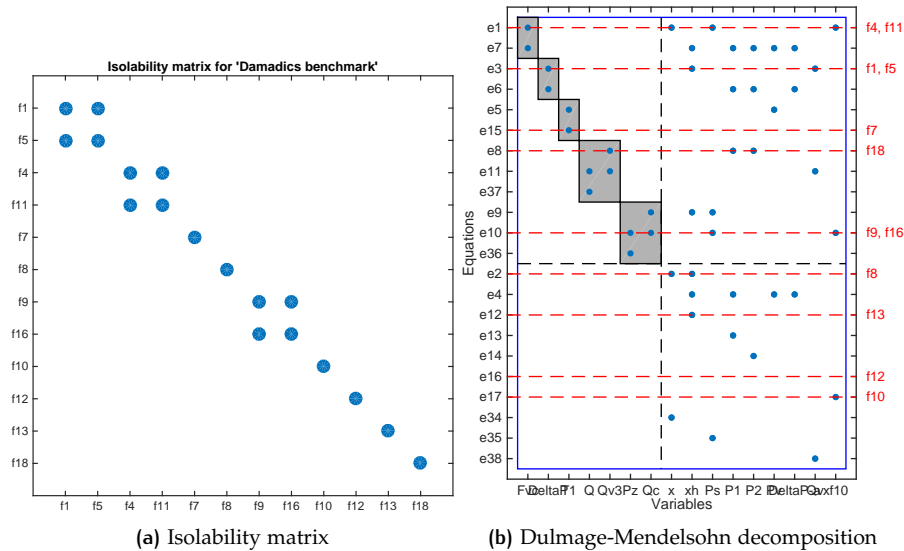


Figure 7: Isolability properties after adding isolability sensors.

## 8 RESIDUAL GENERATOR DESIGN

In the literature, there are many different proposed approaches for residual generation, e.g., based on parity relations, Extended Kalman Filters, adaptive/high-gain/sliding-observers and so on. In this toolbox, two basic approaches are implemented to generate residual generators that are general enough to be automatic and supported by structural analysis. It should be emphasized that these are not to be accepted as the only or best solution, just two approaches that are particularly well suited for automatic code generation. The first is here called sequential residual generation and the second is a differential-algebraic observer technique suitable for low-index problems.

### 8.1 Sequential residual generator design

A sequential residual generator, although the name may not be standard, the basic approach is well known. The basic idea is that, given an overdetermined set of equations, find a computational sequence for the unknown variables, and then verify consistency of the set of equations and observations by inserting the variables into the residual equations. Since dynamic systems are studied, questions arise on how to deal with differential constraints, and in this framework there are two ways; either you integrate or you differentiate. The code generated here can be in so called derivative causality, integral causality, or mixed causality. If there are

reference?

algebraic loops in the computational sequence, the toolbox will use the equation solving capabilities of the Symbolic Math Toolbox in Matlab. Works that describes the basic procedure are [6, 16].

To describe the basic steps of the approach, consider an overdetermined set of equations

$$g_i(x, z, f) = 0, \quad i = 1, \dots, n$$

A first step is to partition the set of equations into an exactly determined, with respect to the unknown variables  $x$ , part and a residual equation part as

$$\begin{aligned} g_i^1(x, z, f) &= 0, \quad i = 1, \dots, n_1 \\ g_i^r(x, z, f) &= 0, \quad i = 1, \dots, n_r \end{aligned}$$

The exactly determined part  $g^1$  is then used to solve for  $x$  and compute  $\hat{x}$ , which is then inserted into the residual equation to compute a residual as

$$r = g_i^r(\hat{x}, z, f) = 0$$

If the model is dynamic, the computational sequence might include differentiations, integrations or both. This is referred to as the sequential residual generator is in derivative, integral, or mixed causality.

The toolbox supports this design methodology using the class methods `Matching` and `SeqResGen`. The method `Matching` computes a computational sequence given an *exactly determined* set of equations. Then, `SeqResGen` is called given a matching and residual equations to generate the code. The examples below generates Matlab code but the toolbox also supports generating C code that is Matlab callable. See Section 8.2 for some further details on C code generation and compiling. As an example, consider the case where the model is an MSO set, i.e., a minimally structurally overdetermined set of equations. This means that by subtracting any equation, what is left is an exactly determined set of equations. To generate a residual, where the first equation in an MSO set is used as residual equation, and the rest is used to compute the unknown variables, the following code can be used

```
1 Gamma = model.Matching(setdiff(mso,mso{1})); % compute matching
2 model.SeqResGen( Gamma, mso{1}, 'ResGen' );
```

which will create the file `ResGen.m`, implementing the residual generator. The generated code corresponds to the function

$$[r_t, x_{t+1}] = \text{ResGen}(z_t, x_t, \theta, 1/f_s)$$

where  $z_t$  and  $x_t$  are the known signals and residual generator state at time point  $t$ ,  $\theta$  the model parameters, and  $f_s$  the sampling rate. See Appendix C.1 for an example from the use case in Section 9. To use the generated function to compute a residual, based on measurements  $z$ , something like this is used

```
1 for k=1:N
2   [r(k), state] = ResGen( z(k,:), state, params, 1/fs );
3 end
```

The state consists of information about variables  $x_i$  that are numerically *integrated* and variables  $x_d$  that are numerically *differentiated* according to

$$\text{state}_t = (x_{i_t}, x_{d_{t-1}})$$

Basic structure of the generated code is shown in the pseudo-code below

```
1 function r_t = ResGen( z_t, state_t, params, T_s )
2   % Initialize parameters
3   theta = params.theta;
```

```

4
5 % Initialize integral state variables
6 (xi_t, xd_{t-1}) = state
7
8 % Residual generator body, sequential computations
9 x1_t = ...
10 ...
11 xd_t = d/dt(.)
12 ...
13 xn_t = ...
14
15 % Compute residual value
16 r_t = ...
17
18 % Integrate state variables
19 xi_{t+1} = xi_t + \int_t^{t+T_s} xi'(\tau) d\tau
20
21 % Update state variables
22 state = (xi_{t+1}, xd_t)
23 end

```

See the use case in Section 9 for further details.

The causality of the residual generator is an important property. For an MSO set with  $n$  equations, with 1 more equation than unknown variables, and where each subset of  $n - 1$  equations is exactly determined, there are  $n$  possible sequential residual generators. Although they are based on the same set of equations, they might have dramatically different properties. One such, important, property is the causality of the residual generator. To investigate, there is a convenience function that automatically computes the causality of the sequential residual generator for each choice of residual equation. The class method `MSOCausalitySweep` is called as

```
1 model.MSOCausalitySweep( mso )
```

which will output `der`, `int`, `mixed`, or `algebraic` for each case. For example, consider an MSO set consisting of 6 equations. A sample output of `MSOCausalitySweep` is then

```
1 >> model.MSOCausalitySweep( mso )
2   'int'   'mixed'   'der'   'mixed'   'der'   'int'
```

This means that using the first equation as a residual equation and the remaining 5 would lead to a sequential residual generator in integral causality. Using the second as residual equation would result in a mixed causality residual generator and so on.

It is possible to explicitly specify how the residual equation shall be interpreted in case it is a differential constraint. The options are `derivative` and `integral` and corresponds to the alternatives

$$r = x - \int x' dt, \quad r = x' - \frac{d}{dt}x$$

where  $x$  and  $x'$  are the variable and the corresponding computed derivative. In Matlab, the key `'diffres'` is given to the method `MSOCausalitySweep` as

```
1 model.MSOCausalitySweep( mso, 'diffres','int' )
2   'int'   'mixed'   'der'   'mixed'   'der'   'int'
```

There is also the possibility to ask for a boolean variable indicating if it is possible to realize a residual generator for a given MSO set in derivative or integral causality respectively. The call looks like

```
1 model.MSOCausalitySweep( mso, 'causality','der' )
```

## 8.2 Generating and compiling generated C code

The toolbox supports generating C code that can be compiled using the mex functionality in Matlab. The call to generate C code is similar to generating Matlab code, only passing the language option to the SeqResGen as

```
1 model.SeqResGen( Gamma, mso{1},'ResGen', 'language', 'C' );
```

This will generate the file ResGen.cc that can be compiled with any C++ compiler<sup>5</sup>. To compile in Matlab, write

```
1 >> mex ResGen.cc
2 Building with 'Xcode Clang++'.
3 MEX completed successfully.
```

This function can be used exactly the same way as the Matlab counterpart. The function generated as above, and in Matlab mode, computes the residual for one sample at a time. In C mode, a batch function can also be generated. In the batch mode, not only observations at one time-point is given to the residual generator, but an entire dataset. The computational speedup is generally significant. To generate the batch version of the residual generator, write

```
1 model.SeqResGen( Gamma, mso{1},'ResGenb', 'language', 'C', 'batch', true );
```

and the file ResGenb.cc is compiled exactly the same way as above.

If the model has external functions, as described in Section 2.2.2, these functions has to be implemented also in C code, compiled, and linked with the generated residual generator code. For example, say that all external functions are implemented in a file externalfuns.cc with a corresponding header file externalfuns.h. Then, with the external option, the generated code will include the external function headers.

```
1 model.SeqResGen( Gamma, mso{1},'ResGen', 'language', 'C', ...
2 'external', 'externalfuns.h' );
```

Then, compiling and linking the whole thing is then done for example by

```
1 >> mex ResGen.cc externalfuns.cc
2 Building with 'Xcode Clang++'.
3 MEX completed successfully.
```

## 8.3 Observer based residual generator design

This approach aims at generating code for an observer that estimates the unknown variables and computes a residual. The residual generator will be formulated as a DAE, which can be integrated using any standard ODE solver which means that it is only applicable to low-index problems [14, 8]. To describe the method, partition the unknown variables  $x$  into  $x_1$  which are state variables, and  $x_2$  which are algebraic variables. Then, the model can be described by

$$\begin{aligned} g_i(dx_1, x_1, x_2, z, f) &= 0 & i = 1, \dots, n \\ dx_1 &= \frac{d}{dt}x_1 & i = 1, \dots, m \end{aligned} \quad (4)$$

<sup>5</sup> The current implementation utilizes *some* C++ functionality and therefore the file need to be compiled with a C++ compiler.

where the dynamic relations has been explicitly described. Important note, the implemented approach is only applicable to models of *low (structural) differential index* [8], i.e., state-space models and implicit state-space models. This restriction is important and note that if MSO sets, or other submodels, are considered when generating residuals, the low-index property is not necessarily fulfilled even though the original model is in state-space form. Loosely, the low-index property corresponds to that there exists, locally, unique solutions for the highest ordered derivatives in  $g_i$ . Let  $g = (g_1, \dots, g_n)$ , then the model (4) is of low index at  $x = x_0$  and  $z = z_0$  if

$$\left( \begin{array}{cc} \frac{\partial g}{\partial dx_1} & \frac{\partial g}{\partial x_2} \end{array} \right) \Big|_{x=x_0, z=z_0}$$

has full column rank. In the toolbox, *structural* low index is verified which corresponds to that there exists a complete matching of the highest ordered derivatives in the equations  $g$ .

$$\begin{aligned} \dot{x}_1 &= g_1(x_1, x_2, z, f) \\ 0 &= g_2(x_1, x_2, z, f), \quad \frac{\partial g_2}{\partial x_2} \text{ is full column rank} \\ 0 &= g_r(x_1, x_2, z, f) \end{aligned} \quad (5)$$

In the toolbox, a test on if the model is structurally high-index or not using the call `IsHighIndex`

```
1 model.IsHighIndex()
```

which tests if there exists a low-index submodel or not. To test if a specified submodel, e.g., an MSO set is high-index, write

```
1 model.IsHighIndex( mso )
```

where `mso` is a vector of equation indices. The method `IsHighIndex` determines if the model has low structural differential index, i.e., 0 or 1. To determine the exact structural index, as could be interesting to analyze in high-index problems, then the method `Pantelides` from [13] can be directly applied.

```
1 strucIndex = model.Pantelides()
```

Note that method `Pantelides` can only be called on exactly determined systems.

For a low-index system, equations (5) is used to form the DAE observer, with a feedback gain  $K(x, z)$  introduced as

$$\begin{aligned} \hat{x}_1 &= g_1(\hat{x}_1, \hat{x}_2, z) + K(\hat{x}, z)g_r(\hat{x}_1, \hat{x}_2, z) \\ 0 &= g_2(\hat{x}_1, \hat{x}_2, z) \end{aligned}$$

This observer estimates the unknown states  $x_1$  and the algebraic variables  $x_2$ . Then, the residual equations in  $g_r$  from (5) can be used to compute the residual.

$$r = g_r(\hat{x}_1, \hat{x}_2, z)$$

To generate code, suitable to be integrated using any solver that is suitable for low-index DAE:s, is generated using the class method `ObserverResGen`

```
1 model.ObserverResGen( mso, 'ResGen' );
```

This call will generate the file `ResGen.m`, see Appendix C.4 for an example from the use case in Section 9. Two possible types of integrators that might be used are solvers based on Backward Differentiation Formula (BDF) or implicit Runge-Kutta (IRK) methods [2]. Possible Matlab solvers are `ode15s` and `ode23t`.

Let an extended state vector be  $w = (\hat{x}_1, \hat{x}_2, r)$ , and the dimensions for  $\hat{x}_1$ ,  $\hat{x}_2$ , and  $r$  respectively be  $n_1$ ,  $n_2$ , and  $n_r$ . The generated code corresponds to the function  $F(w, z)$  in the DAE model

$$M\dot{w} = \begin{pmatrix} g_1(\hat{x}_1, \hat{x}_2, z) + K(\hat{x}, z)g_r(\hat{x}_1, \hat{x}_2, z) \\ g_2(\hat{x}_1, \hat{x}_2, z) \\ r - g_r(\hat{x}_1, \hat{x}_2, z) \end{pmatrix} = F(w, z) \quad (6)$$

where the mass matrix  $M$  is given by

$$M = \begin{pmatrix} I_{n_1} & 0_{n_1 \times (n_2 + n_r)} \\ 0_{(n_2 + n_r) \times n_1} & 0_{(n_2 + n_r) \times (n_2 + n_r)} \end{pmatrix}$$

A DAE model in the form

$$M\dot{w} = f(w)$$

where the mass matrix  $M$  can be integrated using stiff, implicit ODE solvers. For example, the standard Matlab ODE solver `ode15s` can be directly used.

As an example of a function call to use the generated residual generator, let  $z$  and  $t$  be the observations and corresponding time stamps and let  $K$  be a constant observer gain. Then the residual generator can be simulated by:

```

1 M = [eye(2) zeros(2,4); zeros(4,6)];
2 [~,w] = ode15s(@(ts,x) ResGen(x, interp1(t,z,ts), K, params), ...
3   t, xo, odeset('Mass',M));
4 r=w(:,6)

```

where, in this case,  $n_1 = 2$ ,  $n_2 = 3$ , and  $n_r = 1$ .

As with any feedback system, the feedback gain need to be determined to ensure estimator stability. In general, this is a difficult problem but the toolbox can provide some guidance. By supplying operating point and known variables, linearization matrices can be automatically computed. Let

$$A_{i,j} = \left. \frac{\partial g_i}{\partial x_j} \right|_{x=x_0, z=z_0}, \quad i, j = 1, 2$$

$$C_j = \left. \frac{\partial g_r}{\partial x_j} \right|_{x=x_0, z=z_0}, \quad j = 1, 2$$

Let the estimation error be  $e = x_1 - \hat{x}_1$ , then the linearized error dynamics of the observer is  $\dot{e} = (A - KC)e$  with

$$A = (A_{11} - A_{12}A_{22}^{-1}A_{21})$$

$$C = -(C_1 - C_2A_{22}^{-1}A_{21})$$

The low-index property of the model ensures that matrix  $A_{22}$  is invertible. To obtain the matrices  $A$  and  $C$ , add options `linpoint` and `parameters` to the method call like the following

```

1 [A,C] = model.ObserverResGen(mso, 'ResGen', 'linpoint', linpoint, ...
2   'parameters', params);

```

See further details in the use case in Section 9.4 for examples on how to form `linpoint` and `parameters`, and also how to compute a locally stabilizing feedback gain  $K$ .

## 9 USE CASE

This section shows a simple use case how the toolbox can be used. As an example model, the three-tank model from [6] is used. All code is available, and possible to

run, in file `usecase.m` found in the `examples/ThreeTankSimulation/` directory. The same usecase, but with C-generated residual generators is found in file `usecase.c.m` in the same directory. The three-tank system is shown in Figure 8 and simple model

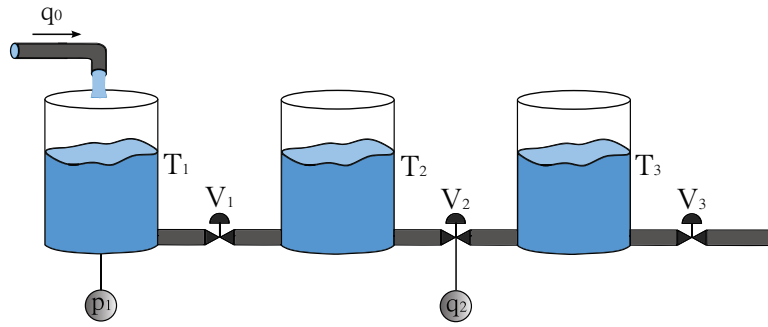


Figure 8: Diagram of the three-tank system.

of the system is

$$\begin{aligned}
 e_1 : q_1 &= \frac{1}{R_{V1}}(p_1 - p_2) & e_7 : y_1 &= p_1 \\
 e_2 : q_2 &= \frac{1}{R_{V2}}(p_2 - p_3) & e_8 : y_2 &= q_2 \\
 e_3 : q_3 &= \frac{1}{R_{V3}}(p_3) & e_9 : y_3 &= q_0 \\
 e_4 : \dot{p}_1 &= \frac{1}{C_{T1}}(q_0 - q_1) & e_{10} : \dot{p}_1 &= \frac{dp_1}{dt} \\
 e_5 : \dot{p}_2 &= \frac{1}{C_{T2}}(q_1 - q_2) & e_{11} : \dot{p}_2 &= \frac{dp_2}{dt} \\
 e_6 : \dot{p}_3 &= \frac{1}{C_{T3}}(q_2 - q_3) & e_{12} : \dot{p}_3 &= \frac{dp_3}{dt}
 \end{aligned}$$

where  $p_i$  is the pressure in tank  $i$ ,  $q_i$  the flow through valve  $i$ ,  $R_{Vi}$  the flow resistance of valve  $i$ , and  $C_{Ti}$  the capacitance of tank  $i$ . Three sensors  $y_1$ ,  $y_2$ , and  $y_3$ , measure  $p_1$ ,  $q_2$ , and  $q_0$ , respectively. For this study, six parametric faults have been considered in the plant: change in capacity of tanks  $C_{T1}$ ,  $C_{T2}$ , and  $C_{T3}$ , and partial blocks in valves  $R_{V1}$ ,  $R_{V2}$ ,  $R_{V3}$ .

### 9.1 Model definition

Here, the model will be defined using symbolic expressions. Therefore, the model is defined using the following Matlab code

```

1 modelDef.type = 'Symbolic';
2 modelDef.x = {'p1','p2','p3','q0','q1','q2','q3','dp1','dp2','dp3'};
3 modelDef.f = {'fV1','fV2','fV3','fT1','fT2','fT3'};
4 modelDef.z = {'y1','y2','y3'};
5 modelDef.parameters = {'Rv1','Rv2','Rv3','CT1','CT2','CT3'};
6
7 syms(modelDef.x{:})
8 syms(modelDef.f{:})
9 syms(modelDef.z{:})
10 syms(modelDef.parameters{:})
11
12 modelDef.rels = {q1==1/Rv1*(p1-p2) + fV1,... % e1
13 q2==1/Rv2*(p2-p3) + fV2, ... % e2
14 q3==1/Rv3*p3 + fV3,... % e3
15 dp1==1/CT1*(q0-q1) + fT1,... % e4

```



```

16 dp2==1/CT2*(q1-q2) + fT2, ... % e5
17 dp3==1/CT3*(q2-q3) + fT3, ... % e6
18 y1==p1, y2==q2, y3==q0,... % e7, e8, e9
19 DiffConstraint('dp1','p1 '),... % e10
20 DiffConstraint('dp2','p2 '),... % e11
21 DiffConstraint('dp3','p3 '),... % e12
22 };
23
24 model = DiagnosisModel( modelDef );
25 model.name = 'Three tank system';

```

With a model object, the model structure can be plotted using the simple command

```
1 model.PlotModel();
```

which will produce Figure 9.

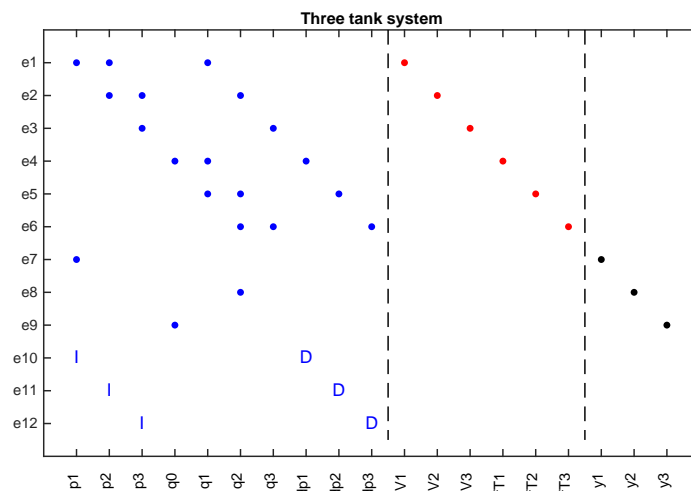


Figure 9: Three tank model structure.

## 9.2 Isolability analysis

Since sensors already have been added in the model definition, one next step is to see what kind of isolability properties that are possible (structurally). To do isolability analysis, as described in [6], the class method `IsolabilityAnalysis` can be used. Here, it is possible to show what isolability that is possible using derivative, integral, or mixed causality residual generators. First, to look at the derivative and integral causality cases, the commands

```

1 model.IsolabilityAnalysis('causality','der');
2 model.IsolabilityAnalysis('causality','int');

```

produces Figures 10-a and b. See [6] for details on how to interpret the figures.

Figure 11-a shows the full structural isolability properties of the model, i.e., performance in mixed causality. The figure tells us that it is possible to 1) uniquely isolate faults  $fV1$ ,  $fT1$ , and  $fT2$ , and 2) the group of faults  $\{fV2, fV3, fT3\}$  can be detected and isolated from the other faults, but can not be separated from each other. Figure 11-b shows the corresponding Dulmage-Mendelsohn decomposition, with indication of faults and canonical decomposition of the overdetermined part of the model. In this case, the model only consist of an overdetermined part. The following commands in Matlab produces the figures

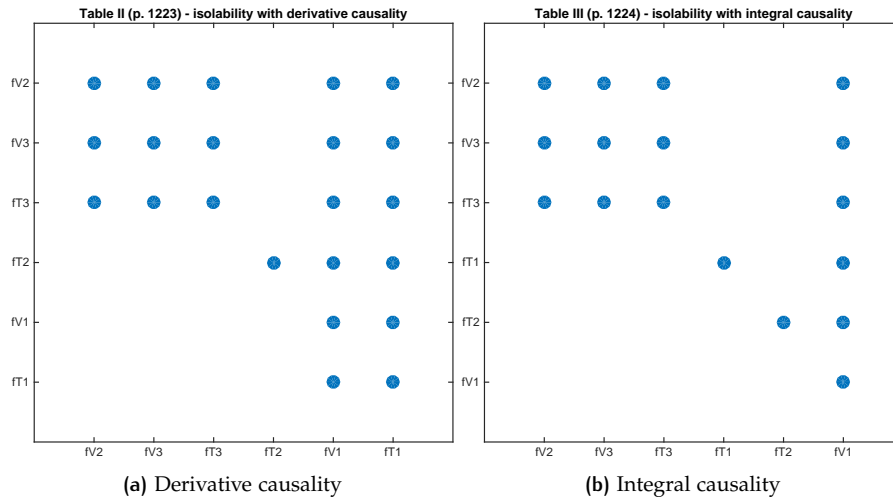


Figure 10: Isolability matrices in derivative and integral causality.

```

1 model.IsolabilityAnalysis ();
2 model.PlotDM('eqclass', true, 'fault', true );

```

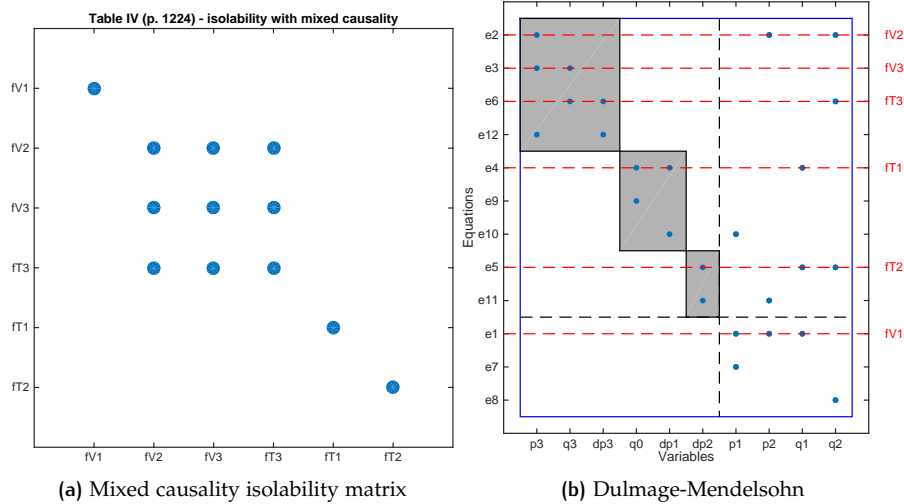


Figure 11: Isolability analysis in the mixed causality case, together with the Dulmage-Mendelsohn decomposition. The figure shows the canonical decomposition of the overdetermined part, with indications where the faults appear in the model.

With the decomposition, the isolability properties of mixed causality case is clearly visible since the faults appear in different equivalence classes, except for the group  $\{fV2, fV3, fT3\}$  which appears in the same class.

### 9.3 Find overdetermined set of equations

Let's say we are happy with the isolability performance in Figure 11, a next step is to design residual generators. One way to do this is to find overdetermined set of equations and use those to design residual generators. For this, we compute the set of MSO sets, i.e., the set of minimally structurally overdetermined sets of equations, see [11] for full details on how this is done. The Matlab command

```

1 msos = model.MSO();

```

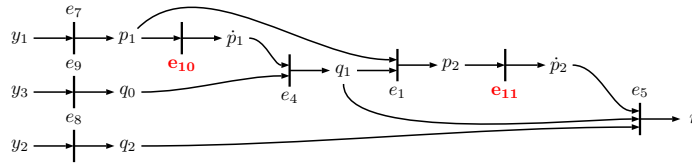


Figure 12: Residual  $r_1$ , sequential residual generator in derivative causality based on MSO set  $\mathcal{M}_2$ .

gives the following 6 MSO sets

$$\begin{aligned}
 \mathcal{M}_1 &= \{e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}\} \\
 \mathcal{M}_2 &= \{e_1, e_4, e_5, e_7, e_8, e_9, e_{10}, e_{11}\} \\
 \mathcal{M}_3 &= \{e_1, e_2, e_3, e_5, e_6, e_7, e_8, e_{12}, e_{11}\} \\
 \mathcal{M}_4 &= \{e_1, e_2, e_3, e_4, e_6, e_7, e_8, e_9, e_{10}, e_{12}\} \\
 \mathcal{M}_5 &= \{e_1, e_2, e_3, e_4, e_5, e_6, e_8, e_9, e_{10}, e_{11}, e_{12}\} \\
 \mathcal{M}_6 &= \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_9, e_{10}, e_{11}, e_{12}\}
 \end{aligned} \tag{7}$$

This means that these 6 are the *minimal* sets of equations that has redundancy and therefore can be used to design residual generators.

#### 9.4 Design residual generators

A next step is to use these overdetermined sets of equations to generate residuals. For demonstration purposes, 4 different designs will be made. The first three are sequential residual generators, one in derivative causality ( $r_1$ ), one in integral causality ( $r_2$ ), and one in mixed causality ( $r_3$ ). The fourth residual generator ( $r_4$ ) will be designed using a simple observer based approach. Residual generators  $r_1$ ,  $r_2$  and  $r_4$  will use MSO set  $\mathcal{M}_2$  in (7) and  $r_3$  will use  $\mathcal{M}_1$ .

The first residual generator corresponds to Fig. 2 in [6], that use equation  $e_5$  as a residual equation and the remaining, exactly determined, equations in  $\mathcal{M}_2$  to compute the unknown variables. Figure 12 shows the corresponding computational graph. It is clear that the residual generator is in derivative causality, since all differential constraints (indicated in red in the figure) computes the differentiated variable from the non-differentiated. To generate code for the residual generator, first the matching is found using all equations but  $e_5$ , then the Matlab code is generated based on this matching. The corresponding Matlab code is

```

1 Gamma1 = model.Matching(setdiff(msos{2},5)); % compute matching
2 model.SeqResGen( Gamma1, 5, 'ResGen1' );

```

and the generated code is shown in Appendix C.1.

If using  $e_7$  instead of  $e_5$  as a residual equation, with the same MSO set, we obtain a residual generator in integral causality instead. The corresponding computational graph, same as Fig. 3 in [6], is shown in Figure 13. It is clear that the residual generator is in integral causality, since all differential constraints computes the integral of a differentiated variable. Code generation is done as before,

```

1 Gamma2 = model.Matching(setdiff(msos{2},7)); % compute matching
2 model.SeqResGen( Gamma2, 7, 'ResGen2' );

```

and the generated code is included in Appendix C.2.

To show an example of a sequential residual generator in mixed causality, i.e., where both differentiation and integration is used in the computation of the residual, consider MSO set  $\mathcal{M}_1$  with equation  $e_2$  as residual equation. The computational graph is shown in Figure 14, which corresponds to Fig. 12 in [6]. Code generation is done again in the same way

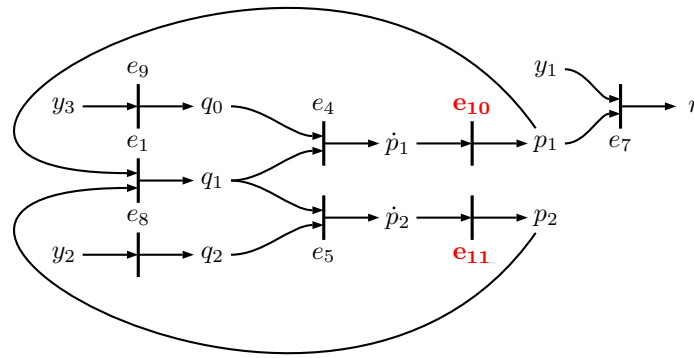


Figure 13: Residual  $r_2$ , sequential residual generator in integral causality based on mso  $\mathcal{M}_2$ .

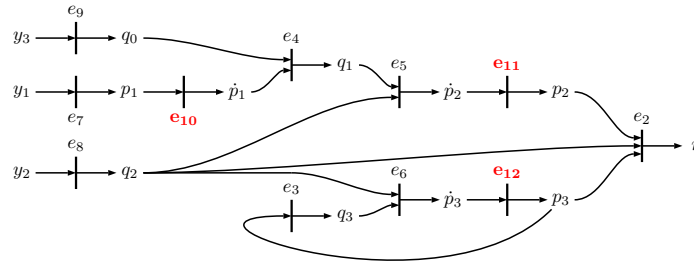


Figure 14: Residual  $r_3$ , sequential residual generator in mixed causality based on mso  $\mathcal{M}_1$ .

```

1 Gamma3 = model.Matching(setdiff(msos{1},2)); % compute matching
2 model.SeqResGen( Gamma3, 2, 'ResGen3' );

```

and the resulting code is shown in Appendix C.3.

In the fourth residual generator, mso  $\mathcal{M}_2$  is again used but now using an observer approach. Then, no residual equation need to be specified, the approach chooses residual equations automatically. Note that this approach is only applicable to low-index problems. If a linearization point and values on the parameters are provided,  $A$  and  $C$  matrices are computed such that a feedback gain can be designed to (locally) stabilize the observer. In code, this corresponds to

```

1 linpoint.xo = [0,0,0,0,0,0]; linpoint.zo = [0;0;0];
2 params.Rv1 = 1; params.Rv2 = 1; params.Rv3 = 1;
3 params.CT1 = 1; params.CT2 = 1; params.CT3 = 1;
4 [A,C] = model.ObserverResGen( msos{2}, 'ResGen4', 'linpoint', linpoint, ...
5 'parameters', params );

```

and the generated code is included in Appendix C.4. To find a feedback gain  $K$  using pole placement, a simple approach is for example

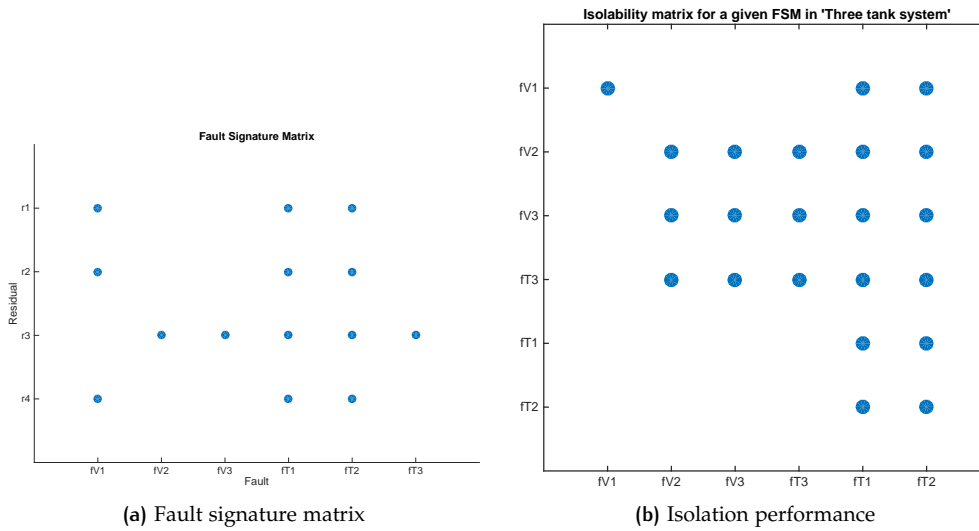
```

1 K = place(A',C',[-0.2,-0.3]);

```

## 9.5 Isolability properties of residual generators

The isolability analysis in Section 9.2 was done for the model, not these four particular residual generators. This can be done using the class methods `FSM` and `IsolabilityAnalysisFSM`. First, the fault signature matrix (FSM), i.e., which faults each residual is (structurally) sensitive to is determined using `FSM`, and then this fault signature matrix is used to compute the isolability properties using `IsolabilityAnalysisFSM`. The results are shown in Figure 15 and the figures are generated using the commands



**Figure 15:** Fault signature matrix (FSM) and isolability properties for the four residuals  $r_1, \dots, r_4$ .

```

1 FSM = model.FSM({msos{2}, msos{2}, msos{1}, msos{2}});
2 spy(FSM,30)
3 set(gca, 'YTick', 1:4, 'XTick', 1:model.nf,...
4 'YTickLabel', {'r1', 'r2', 'r3', 'r4'}, 'XTickLabel',model.f, 'box', 'off');
5 xlabel(' Fault')
6 ylabel(' Residual')
7 title(' Fault Signature Matrix')
8
9 model.IsolabilityAnalysisFSM( FSM );

```

Figure 15-b should be compared to Figure 11-a and it is clear that the four residuals chosen does not reach the best possible isolation performance. This is not surprising since only MSO sets  $\mathcal{M}_1$  and  $\mathcal{M}_2$  were used to generate the four residuals. If, for example residual generators for  $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$ , and  $\mathcal{M}_4$  were to be designed, full structural isolability would be achieved.

## 9.6 Simulation results

The system can now be simulated in all fault modes. For the simulations, a simple LQ-controller is designed such that the level in tank 1 follows a reference signal. A sample, noise-free, simulation result for the no-fault case is shown in Figure 16. A simulation of the faulty case  $f_{Rv1}$  is shown in Figure 17 where a ramp fault is added, starting at  $t = 6$  and reaching top value at  $t = 10$ .

To simulate the residual generators, residuals  $r_1, r_2$ , and  $r_3$  are implemented in discrete time. Let  $z$  and  $t$  be the observations and time stamps respectively, then simulation of the residual generators is done by

```

1 for k=1:N
2 [r1(k), state1] = ResGen1( z(k,:), state1, params, 1/fs );
3 [r2(k), state2] = ResGen2( z(k,:), state2, params, 1/fs );
4 [r3(k), state3] = ResGen3( z(k,:), state3, params, 1/fs );
5 end

```

Residual generator  $r_4$  was created in continuous time, and therefore one of Matlab's standard ODE integrators are used. The code corresponds to a dynamic system in the form (6) and note that a stiff, implicit, solver is needed to integrate the

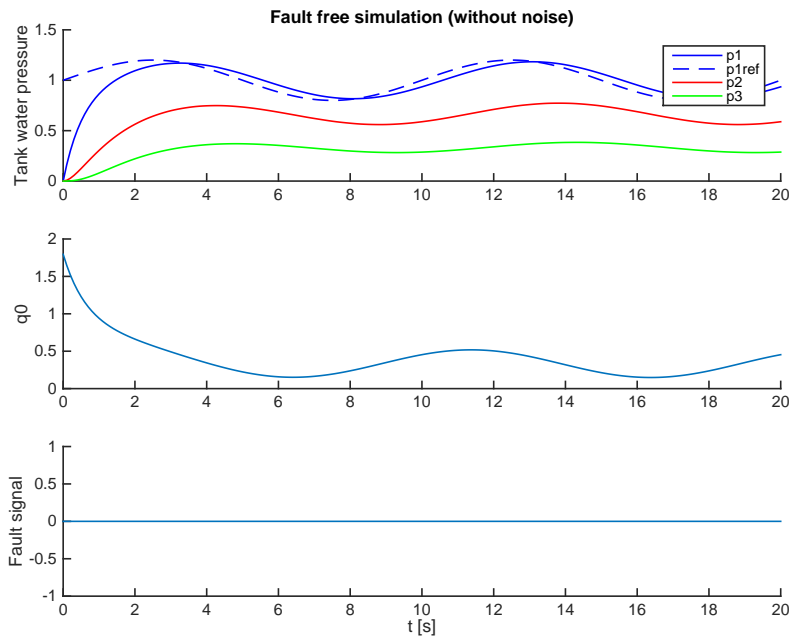


Figure 16: Simulation of fault free operation of the three-tank system.

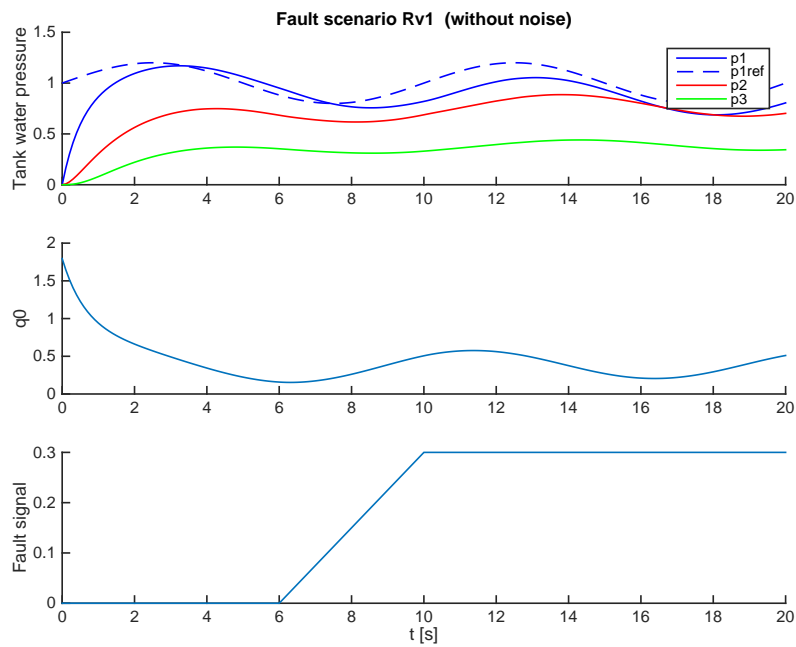


Figure 17: Simulation of the faulty case,  $f_{Rv1}$ , of the three-tank system.

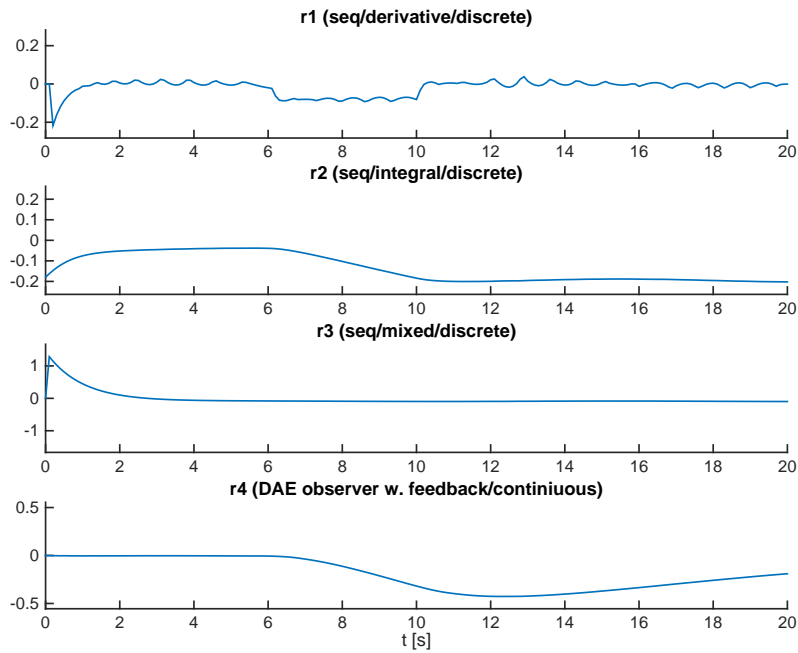
observer. In our case, we chose the solve ode15s. The code to integrate the residual generator is

```

1 M4 = [eye(2) zeros(2,4); zeros(4,6)];
2 [~,x] = ode15s(@(ts,x) ResGen4(x, interp1(t,z,ts), K4, params), ...
3   t, x0, odeset('Mass',M4, 'AbsTol', 1e-3));
4 r4 = x(:,6);

```

where we use interp1 to interpolate measurement values in between sampling points. For this fault mode, residuals  $r_1$ ,  $r_2$ , and  $r_4$  shall indicate a fault while  $r_3$  shall not, see Figure 15. Figure 18 shows the residuals. It is clear that, after the



**Figure 18:** Residual values for residuals  $r_1$ ,  $r_2$ ,  $r_3$ , and  $r_4$  for the fault mode  $f_{Rv1}$  shown in Figure 17.

initial transient all residuals react as expected. It is also clear that  $r_1$  is more noisy and that is due to that this residual is in derivative causality and the approximate differentiation introduces this error. It is also clearly visible that residual  $r_1$  only reacts to changes in the fault, i.e., only reacts in the interval  $[6, 10]$ . Remember that residuals  $r_2$  and  $r_4$  are based on exactly the same set of equations as  $r_1$  and this clearly shows how different ways of realizing a residual generator, based on the same set of equations has different performance and properties.

## APPENDIX

## A SUMMARY OF CLASS METHODS

Below is a table that summarizes all the class methods for `DiagnosisModel` objects. The description is brief and the most complete documentation can be found by starting the documentation browser using `doc DiagnosisModel`, or accessing the help documentation by

help DiagnosisModel.methodName	
Method name	Description
<a href="#">Model object definition and manipulation</a>	
<code>DiagnosisModel</code>	Constructor for model object used for diagnosis analysis and residual generator code generation
<code>AddSensors</code>	Add sensors to a model
<code>AddEquations</code>	Add equations to a model
<code>RemoveFaultVariables</code>	Remove fault variables from a model
<code>ReplaceEquations</code>	Replace equations in a model
<code>LumpDynamics</code>	Lump dynamic variables for structural model
<code>Structural</code>	Convert a symbolic model to a structural model
<code>copy</code>	Make a new copy of the model object
<a href="#">Model properties</a>	
<code>x</code>	Unknown variables
<code>f</code>	Fault variables
<code>z</code>	Known variables
<code>X</code>	Incidence matrix for unknown variables
<code>F</code>	Incidence matrix for fault variables
<code>Z</code>	Incidence matrix for known variables
<code>e</code>	Equation names
<code>name</code>	Model name
<code>type</code>	Model type
<code>P</code>	List of possible sensor locations
<code>Pfault</code>	Which sensor locations may be faulty
<code>parameters</code>	List of model parameter names
<code>syms</code>	Symbolic equations
<a href="#">Model exploration</a>	
<code>Lint</code>	Print model information and check for inconsistencies
<code>PlotDM</code>	Plots Dulmage-Mendelsohn decomposition of model structure
<code>PlotModel</code>	Plots a model object
<code>SubModel</code>	Extract submodel
<code>MeasurementEquations</code>	Extract indices to measurement equations
<code>DifferentialConstraints</code>	Extract indices to differential constraints and variables in the model
<code>GenSimulationModel</code>	Generate code for simulation model, usable directly with Matlabs ODE/DAE solvers. Model must be low-index and exactly determined.
<code>ne</code>	Number of equations in model
<code>nf</code>	Number of fault variables in model
<code>nx</code>	Number of unknown variables in model

*continued on next page*



<i>continued from previous page</i>	
Method name	Description
nz	Number of known variables in model
Redundancy	Compute the structural degree of redundancy of a model
MTESRedundancy	Compute the structural degree of redundancy for an MTES set
srank	Compute the structural rank of the incidence matrix for the unknown variables
AlgebraicVariables	Extract the algebraic variables of the model
DynamicVariables	Extract the dynamic variables of the model
IsStatic	Is the set of model equations static?
IsDynamic	Is the set of model equations dynamic?
IsHighIndex	Is the model of high structural differential index?
IsLowIndex	Is the model of low structural differential index?
IsPSO	Is the model proper structurally overdetermined?
IsObservable	Is the (sub-)model structurally observable?
Pantelides	Run Pantelides algorithm for determining structural index and differentiation vector
BipartiteToLaTeX	Generate a LaTeX document with the bipartite graph corresponding to the structural model
<b>Diagnosability analysis</b>	
DetectabilityAnalysis	Performs a structural detectability analysis
IsolabilityAnalysis	Perform structural single fault isolability analysis of model
IsolabilityAnalysisArrs	Perform structural single fault isolability analysis of a set of ARRs
IsolabilityAnalysisFSM	Perform structural single fault isolability analysis of a Fault Signature Matrix (FSM)
<b>Sensor placement</b>	
PossibleSensorLocations	Set possible sensor locations
SensorLocationsWithFaults	Set possible sensor locations that has faults in new sensors
SensorPlacementDetectability	Determine minimal set of sensors to achieve detectability
SensorPlacementIsolability	Determine minimal set of sensors to achieve maximal fault isolability
CompiledMHS	Use the compiled minimal-hitting set algorithm if available (not recommended)
<b>Overdetermined equations</b>	
MSO	Compute the set of MSO sets
MTES	Computes the set of minimal test equation support
FSM	Compute the fault signature matrix (FSM)
TestSelection	A minimal hitting set based test selection approach
CompiledMSO	Use the compiled MSO algorithm if available
<b>Residual generation</b>	
ObserverResGen	Generate Matlab code for observer based residual generator

*continued on next page*

<i>continued from previous page</i>	
Method name	Description
Matching	Compute a matching in the model for a set of equations
PlotMatching	Plot a matching
SeqResGen	Generate Matlab code for sequential residual generator
MSOCausalitySweep	For a given MSO set, determine causality for sequential residual generator for each n residual equations

## B COMPILE C++ IMPLEMENTATIONS

To compile the C++ sources, it is required that a functioning compiler is installed and configured for use with Matlab. Instructions below are directly applicable for Unix/MacOS systems. There are binaries included in the distribution if you do not want to compile the sources yourself. In the directory `binaries` there are compiled files for 64 bit Linux and MacOS. Copy the files corresponding to your system into the `src` directory and hope for the best.

### B.1 Minimal Hitting Set

There is a compiled C++ implementation of the minimal hitting set algorithm used. To compile, open Matlab and go to `src` directory. To compile, type (output from an MacOS system)

```

1 >> mex MHScompiled.cc
2 Building with 'Xcode Clang++'.
3 MEX completed successfully.
```

Verify that everything has worked by

```

1 >> exist('MHSCompiled')
2
3 ans =
4
5     3
```

The use of the compiled algorithm is optional, full functionality is obtained with the Matlab implementation of the minimal hitting set algorithm.

### B.2 MSO algorithm

The MSO algorithm uses a library for computing with sparse matrices. The sparse matrix library is part of the software SuiteSparse and can be downloaded from <http://faculty.cse.tamu.edu/davis/suitesparse.html> and is described in [3]. You do not have to compile and install the entire SuiteSparse library, only the CSparse part. The CSparse source is included in the zip-archive.

To install on a Linux or a Mac with MacOS with developer tools installed, go to the CSparse directory and write<sup>6</sup> at a terminal prompt, not in Matlab:

```

1 > cd CSparse
2 > make
```

<sup>6</sup> Remark for Linux users: On my Linux system, I had to add the compiler flag `-fPIC` on the `CF` line in file `SuiteSparse/CSparse/Lib/Makefile`.

When the CSpase library is compiled, to compile the MSO algorithm, open Matlab and go to the src directory and type:

```

1 %% Specify installation directory for CSpase
2 CPARSEDIR = '../CSpase';
3
4 % Derive include and lib directory
5 CPARSEINC=['-I' fullfile(CPARSEDIR, '/Include')];
6 CPARSELIB=['-L' fullfile(CPARSEDIR, '/Lib')];
7
8 %% Compile sources and link mex-file
9
10 mex('-c', '-largeArrayDims', CPARSEINC, 'MSOAlg.cc');
11 mex('-c', '-largeArrayDims', CPARSEINC, 'SparseMatrix.cc');
12 mex('-c', '-largeArrayDims', CPARSEINC, 'StructuralAnalysisModel.cc');
13 mex('-c', '-largeArrayDims', CPARSEINC, 'FindMSOcompiled.cc');
14
15 % Link
16 mex(CPARSELIB, 'FindMSOcompiled.o', 'SparseMatrix.o', 'StructuralAnalysisModel.o', 'MSOAlg.o', '-lcsparse');
17
18 %% Specify installation directory for CSpase
19 CPARSEDIR = '[INSTALLATIONDIR]';
20
21 % Derive include and lib directory
22 CPARSEINC=['-I' fullfile(CPARSEDIR, '/Include')];
23 CPARSELIB=['-L' fullfile(CPARSEDIR, '/Lib')];
24
25 % Compile sources
26 mex('-c', '-largeArrayDims', CPARSEINC, 'MSOAlg.cc');
27 mex('-c', '-largeArrayDims', CPARSEINC, 'SparseMatrix.cc');
28 mex('-c', '-largeArrayDims', CPARSEINC, 'StructuralAnalysisModel.cc');
29 mex('-c', '-largeArrayDims', CPARSEINC, 'FindMSOcompiled.cc');
30
31 % Link
32 mex(CPARSELIB, 'FindMSOcompiled.o', 'SparseMatrix.o', ...
33 'StructuralAnalysisModel.o', 'MSOAlg.o', '-lcsparse');

```

On a Mac system, you might get warnings when linking due to different deployment versions used by the Matlab compiler and the compiler used, *"ld: warning: object file (./CSpase/Lib/libcspase.a(cs\_add.o)) was built for newer OSX version (10.12) than being linked (10.9)"*. To avoid this warning, recompile CSpase with the build version of Matlab by setting the environment variable `MACOSX_DEPLOYMENT_TARGET=10.9` before running `make` to build CSpase.

Verify that everything has worked and the binary has been generated by

```

1 >> exist('FindMSOcompiled')
2
3 ans =
4
5     3

```

## C GENERATED CODE IN USE-CASE

Below are the generated code for the four residual generators in the use case in Section 9.

### c.1 ResGen1

```

function [r, state] = ResGen1(z,state,params,Ts)
% RESGEN1 Sequential residual generator for model 'Three tank system'
% Causality: Der
%
% Structurally sensitive to faults: fV1, fT1, fT2
%
% Example of basic usage:
% Let z be the observations and N the number of samples, then
% the residual generator can be simulated by:
%
% for k=1:N
%     [r(k), state] = ResGen1( z(k,:), state, params, 1/fs );
% end
% where state is a structure with the state of the residual generator.
% The state has fieldnames: p1, p2

% File generated 29-Feb-2016 16:22:07

% Parameters
Rv1 = params.Rv1;
CT1 = params.CT1;
CT2 = params.CT2;

% Known variables
y1 = z(1);
y2 = z(2);
y3 = z(3);

% Initialize state variables
p1 = state.p1;
p2 = state.p2;

% Residual generator body
q2 = y2; %% e8
q0 = y3; %% e9
p1 = y1; %% e7
dp1 = ApproxDiff(p1,state.p1,Ts); %% e10
q1 = q0-CT1*dp1; %% e4
p2 = p1-Rv1*q1; %% e1
dp2 = ApproxDiff(p2,state.p2,Ts); %% e11

r=dp2-(q1-q2)/CT2; %% e5

% Update state variables
state.p1 = p1;
state.p2 = p2;
end

function dx=ApproxDiff(x,xold,Ts)
if length(xold)==1
    dx = (x-xold)/Ts;
elseif length(xold)==2
    dx = (3*x-4*xold(1)+xold(2))/2/Ts;
else
    error('Differentiation of order higher than 2 not supported');
end
end

```

## c.2 ResGen2

```

function [r, state] = ResGen2(z,state,params,Ts)
% RESGEN2 Sequential residual generator for model 'Three tank system'
% Causality: Int
%
% Structurally sensitive to faults: fV1, fT1, fT2
%
% Example of basic usage:
% Let z be the observations and N the number of samples, then
% the residual generator can be simulated by:
%
% for k=1:N
%   [r(k), state] = ResGen2( z(k,:), state, params, 1/fs );
% end
% where state is a structure with the state of the residual generator.
% The state has fieldnames: p2, p1

% File generated 29-Feb-2016 16:22:09

% Parameters
Rv1 = params.Rv1;
CT1 = params.CT1;
CT2 = params.CT2;

% Known variables
y1 = z(1);
y2 = z(2);
y3 = z(3);

% Initialize state variables
p2 = state.p2;
p1 = state.p1;

% Residual generator body
q2 = y2; %% e8
q0 = y3; %% e9
q1 = (p1-p2)/Rv1; %% e1
dp2 = (q1-q2)/CT2; %% e5
dp1 = (q0-q1)/CT1; %% e4

r=-p1+y1; %% e7

% Update integrator variables
p2 = ApproxInt(dp2,state.p2,Ts); %% e11
p1 = ApproxInt(dp1,state.p1,Ts); %% e10

% Update state variables
state.p2 = p2;
state.p1 = p1;
end

function x1=ApproxInt(dx,x0,Ts)
x1 = x0 + Ts*dx;
end

```

## c.3 ResGen3

```

function [r, state] = ResGen3(z,state,params,Ts)
% RESGEN3 Sequential residual generator for model 'Three tank system'
% Causality: Mixed

```

```

%
% Structurally sensitive to faults: fV2, fV3, fT1, fT2, fT3
%
% Example of basic usage:
% Let z be the observations and N the number of samples, then
% the residual generator can be simulated by:
%
% for k=1:N
%     [r(k), state] = ResGen3( z(k,:), state, params, 1/fs );
% end
% where state is a structure with the state of the residual generator.
% The state has fieldnames: p3, p2, p1

% File generated 29-Feb-2016 16:22:12

% Parameters
Rv2 = params.Rv2;
Rv3 = params.Rv3;
CT1 = params.CT1;
CT2 = params.CT2;
CT3 = params.CT3;

% Known variables
y1 = z(1);
y2 = z(2);
y3 = z(3);

% Initialize state variables
p3 = state.p3;
p2 = state.p2;
p1 = state.p1;

% Residual generator body
q2 = y2; %% e8
q0 = y3; %% e9
q3 = p3/Rv3; %% e3
dp3 = (q2-q3)/CT3; %% e6
p1 = y1; %% e7
dp1 = ApproxDiff(p1,state.p1,Ts); %% e10
q1 = q0-CT1*dp1; %% e4
dp2 = (q1-q2)/CT2; %% e5

r=q2-(p2-p3)/Rv2; %% e2

% Update integrator variables
p3 = ApproxInt(dp3,state.p3,Ts); %% e12
p2 = ApproxInt(dp2,state.p2,Ts); %% e11

% Update state variables
state.p3 = p3;
state.p2 = p2;
state.p1 = p1;
end

function dx=ApproxDiff(x,xold,Ts)
if length(xold)==1
    dx = (x-xold)/Ts;
elseif length(xold)==2
    dx = (3*x-4*xold(1)+xold(2))/2/Ts;
else
    error('Differentiation of order higher than 2 not supported');
end

```

```

    end
end

function x1=ApproxInt(dx,x0,Ts)
    x1 = x0 + Ts*dx;
end

```

#### c.4 ResGen4

```

function dx = ResGen4(x,z,K,params)
% RESGEN4 Observer based residual generatorfor model 'Three tank system'
%
% Structurally sensitive to faults: fV1, fT1, fT2
%
% Example of basic usage:
%   Let z and t be the observations and corresponding timestamps. Let K be the observer gain,
%   then the residual generator can be simulated by:
%
%   [~,x] = ode15s(@(ts,x) ResGen4( x, interp1(t,z,ts), K, params ), t, x0, odeset('Mass',M));
%
%   where the mass matrix M is [eye(2) zeros(2,4);zeros(4,6)]
%   The residual after integration is then r=x(:,6)

% File generated 29-Feb-2016 16:22:13

% Parameters
Rv1 = params.Rv1;
CT1 = params.CT1;
CT2 = params.CT2;

% Known variables
y1 = z(1);
y2 = z(2);
y3 = z(3);

% Model variables
p1 = x(1);
p2 = x(2);
q0 = x(3);
q1 = x(4);
q2 = x(5);
r1 = x(6);

% Algebraic equations
g21 = -q0+y3;
g22 = q1-(p1-p2)/Rv1;
g23 = -q2+y2;

% Residual equations
gr1 = p1+r1-y1;

% Dynamics, with feedback
dp1 = (q0-q1)/CT1 + K(1,:)*r1;
dp2 = (q1-q2)/CT2 + K(2,:)*r1;

% Return value
dx = [dp1; dp2; g21; g22; g23; gr1];
end

```

## D INDEX OF KEYWORDS AND METHODS

### KEYWORDS

- algebraic loops, 19
- aminc, *see* minimal hitting set
- analytical redundancy relations, *see* ARR
- ARR, 15
- causality, 20
  - derivative, *see* derivative causality
  - integral, *see* integral causality
  - mixed, *see* mixed causality
- computational sequence, *see* matching
- conditional constraints, 12
- DAE
  - model, 22
  - observer, 22
- derivative causality, 15, 18
- detectability analysis, 15
- detectable faults, 15
- diagnosability, 15
- DiagnosisModel, 4
- differential constraint
  - residual equation, 20
- differential constraints, 9, 12
- differential index, 11, 22
  - high, 22
  - low, 22
  - structural, 22
- dmperm, *see* Dulmage-Mendelsohn decomposition
- doc, 4
- downloading, 5
- Dulmage-Mendelsohn decomposition, 12
- dynamic model, 9
- dynamics, 6
- equivalence class, 13
- external functions, 12, 21
- fault sensitivity matrix, *see* FSM
- FSM, 15
- handle class, 9
- if-equations, 12
- incidence matrix, 7
- installation, 5
- integral causality, 15, 18
- isolability analysis, 15, 25
  - diagnosis system, 28
- license, 6
- lumped dynamics, 6
- mass matrix, 23
- matching, 19, 27
- MatrixStruc, 7
- methods, 4
- mex, 34
- MHS, *see* minimal hitting set
- minimal hitting set, 16
- minimal hitting set, 34
  - approximate, 16
- minimal test equation support, *see* MTES
- minimally structurally overdetermined, *see* MSO
- mixed causality, 15, 18
- model
  - dynamic, 9
  - structure, *see* structural model
  - symbolic, *see* symbolic model, 11
- model parameters, 11
- MSO, 13
- MTES, 13, 14
- non-detectable faults, 15
- observer gain, 23
- ODE solver, 23
- overdetermined equations, 13, 26
- pole placement, 28
- PSO
  - decomposition, 13
- rels, 8
- residual equation, 19
- residual generator, 18
  - observer, 21, 28
  - sequential, 18
- sensor fault, 17
- sensor placement, 16
- sensor selection, 16
- SeqResGen
  - batch, 21
  - C code generation, 21
- structural model, 6, 7
- symbolic, 11
- symbolic model, 6
- symbolic models, 11
- type, 7, 8
- VarStruc, 8



## METHODS

AddEquations, 32  
AddSensors, 17, 32  
AlgebraicVariables, 33  
  
BipartiteToLaTeX, 9, 33  
  
CompiledMHS, 33  
CompiledMSO, 33  
copy, 9, 32  
  
DetectabilityAnalysis, 15, 33  
DiagnosisModel, 4, 7, 9, 32  
DiffConstraint, 10, 12  
DifferentialConstraints, 32  
DynamicVariables, 33  
  
e, 32  
  
F, 32  
f, 32  
FSM, 15, 28, 33  
  
GenSimulationModel, 32  
GetDMParts, 12  
  
IsDynamic, 33  
IsHighIndex, 22, 33  
IsLowIndex, 33  
IsObservable, 33  
IsolabilityAnalysis, 15, 25, 33  
IsolabilityAnalysisArrs, 15, 33  
IsolabilityAnalysisFSM, 15, 28, 33  
IsPSO, 33  
IsStatic, 33  
  
Lint, 8, 32  
LumpDynamics, 32  
  
Matching, 19, 27, 34  
MeasurementEquations, 32  
MSO, 13, 33  
MSOCausalitySweep, 20, 34  
MTES, 13, 14, 33  
  
MTESRedundancy, 33  
  
name, 32  
ne, 32  
nf, 32  
nx, 32  
nz, 33  
  
ObserverResGen, 22, 28, 33  
  
P, 32  
Pantelides, 22, 33  
parameters, 32  
Pfault, 32  
PlotDM, 5, 32  
PlotMatching, 34  
PlotModel, 8, 10, 32  
PossibleSensorLocations, 16, 33  
PSODecomposition, 13  
  
RandomForestTestSelection, 16  
Redundancy, 33  
RemoveFaultVariables, 32  
ReplaceEquations, 32  
  
SensorLocationsWithFaults, 17, 33  
SensorPlacementDetectability, 17, 33  
SensorPlacementIsolability, 17, 33  
SeqResGen, 19, 21, 27, 34  
srank, 33  
Structural, 32  
SubModel, 32  
syme, 32  
  
TestSelection, 16, 33  
type, 32  
  
X, 32  
x, 32  
  
Z, 32  
z, 32

## REFERENCES

- [1] Mogens Blanke, Michel Kinnaert, Jan Lunze, and Marcel Staroswiecki. *Diagnosis and fault-tolerant control*. Springer, 3rd edition, 2016.
- [2] K. Brenan, S. Campbell, and L. Petzold. *Numerical solution of initial-value problems in differential-algebraic equations*, volume 14. Siam, 1996.
- [3] Timothy A Davis. *Direct methods for sparse linear systems*, volume 2. Siam, 2006.
- [4] Andrew L Dulmage and Nathan S Mendelsohn. Coverings of bipartite graphs. *Canadian Journal of Mathematics*, 10(4):516–534, 1958.
- [5] Dilek Dustegör, Erik Frisk, Vincent Coquempot, Mattias Krysander, and Marcel Staroswiecki. Structural analysis of fault isolability in the DAMADICS benchmark. *Control Engineering Practice*, 14(6):597–608, 2006.
- [6] Erik Frisk, Anibal Bregon, Jan Åslund, Mattias Krysander, Belarmino Pulido, and Gautam Biswas. Diagnosability analysis considering causal interpretations for differential constraints. *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on*, 42(5):1216–1229, 2012.
- [7] Erik Frisk and Mattias Krysander. Residual selection for consistency based diagnosis using machine learning models. In *IFAC SafeProcess*, Warsaw, Poland, August 2018.
- [8] C William Gear and Linda R Petzold. Ode methods for the solution of differential/algebraic systems. *SIAM Journal on Numerical Analysis*, 21(4):716–728, 1984.
- [9] Mattias Krysander, Jan Åslund, and Erik Frisk. A structural algorithm for finding testable sub-models and multiple fault isolability analysis. 21st International Workshop on Principles of Diagnosis (DX-10), Portland, Oregon, USA, 2010.
- [10] Mattias Krysander and Erik Frisk. Sensor placement for fault diagnosis. *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on*, 38(6):1398–1410, 2008.
- [11] Mattias Krysander, Jan Åslund, and Mattias Nyberg. An efficient algorithm for finding minimal overconstrained subsystems for model-based diagnosis. *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on*, 38(1):197–206, 2008.
- [12] J. Armengol Llobet, A. Bregon, T. Escobet, E. R. Gelso, M. Krysander, M. Nyberg, X. Olive, B. Pulido, and L. Trave-Massuyes. Minimal structurally over-determined sets for residual generation: A comparison of alternative approaches. In *Proceedings of IFAC Safeprocess'09*, Barcelona, Spain, 2009.
- [13] Constantinos C Pantelides. The consistent initialization of differential-algebraic systems. *SIAM Journal on Scientific and Statistical Computing*, 9(2):213–231, 1988.
- [14] Linda Petzold. Differential/algebraic equations are not ode's. *SIAM Journal on Scientific and Statistical Computing*, 3(3):367–384, 1982.
- [15] Albert Rosich, Erik Frisk, Jan Åslund, Ramon Sarrate, and Fatiha Nejari. Fault diagnosis based on causal computations. *IEEE Transactions on Systems, Man, and Cybernetics – Part A: Systems and Humans*, 42(2):371–381, 2012.
- [16] Carl Svärd and Mattias Nyberg. Residual generators for fault diagnosis using computation sequences with mixed causality applied to automotive systems. *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on*, 40(6):1310–1328, 2010.

- [17] Carl Svård, Mattias Nyberg, and Erik Frisk. Realizability constrained selection of residual generators for fault diagnosis with an automotive engine application. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 43(6):1354–1369, 2013.
- [18] Carl Svård, Mattias Nyberg, Erik Frisk, and Mattias Krysander. Automotive engine FDI by application of an automated model-based and data-driven design methodology. *Control Engineering Practice*, 21(4):455–472, 2013.